A NEW METHOD FOR TOTAL ELECTRON CONTENT FORECASTING USING GLOBAL POSITIONING SYSTEM MEASUREMENTS

S. M. Stankov (1,3) , **I. S. Kutiev** (1,4) , **N. Jakowski** (2) , **A. Wehrenpfennig** (2)

(1) Geophysical Institute, Bulgarian Academy of Sciences, Sofia 1113, Bulgaria, Email: sstankov@geophys.bas.bg
(2) DLR/Institute of Communication and Navigation, D-17235 Neustrelitz, Germany, Email: Norbert.Jakowski@dlr.de
(3) Royal Meteorological Institute of Belgium, B-1180 Brussels, Belgium, Email: sstankov@oma.be
(4) Institute of Space and Astronautical Science, Sagamihara, 229-8510, Japan, Email: ikutiev@bochan.ted.isas.ac.jp

ABSTRACT

The GPS-derived TEC has proved to be a robust characteristic best representing the ionospheric state during disturbed geomagnetic conditions. Long and short-term TEC forecasting is an important need in the communication and navigation practice. Presented is a new forecasting method consisting of two major parts: (i) TEC monthly median extrapolation (up to 15 days ahead) using Fourier series approximation based on actual data from the past 12 months. (ii) Forecast of the relative deviations of measured TEC from its median values (up to 24 hours ahead) using the Kp index and adjusted through an auto-correlation procedure. Preliminary tests show a good agreement between measured and predicted values.

1. INTRODUCTION

The Total Electron Content (TEC) is a robust characteristic for investigation of the ionosphere-plasmasphere behaviour under both quiet and disturbed conditions. Instantaneous maps of TEC and other ionospheric characteristics are used mostly in the management and optimisation of the high-frequency radio and remote sensing systems, for satellite navigation, development and evaluation of ionosphere-plasmasphere models, etc. Therefore, a TEC short-term forecasting model, based on regular and reliable Global Positioning System (GPS) observations, can be utilized to improve the telecommunication and navigation practice and could be considered in various space weather applications. Various approaches have been used to model and predict TEC: empirical, theoretical, neural networks. Recently, auto- and cross-correlation procedures have been developed [1,2,3,4] for predicting the critical frequency and proving that a short-term forecast should be bound to the geomagnetic activity.

Presented are preliminary results from developing a new method for a single-site TEC forecast based on GPS measurements of the content and on solar/geomagnetic activity indices. If such a forecast is made at several locations in a given area, instantaneous maps can be constructed for the whole region.

2. DATA BASE

Two basic types of measurements are required for the method -- GPS-derived TEC and solar-geomagnetic activity data.

2.1 GPS - derived Total Electron Content

The development database was built on the GPS TEC time-series data acquired at DLR/IKN Neustrelitz in 1995-2001, covering low, rising and top solar activity. After determining the electron content along a number of ray paths by using a special calibration technique for the ionospheric delay of GPS signals, the slant TEC is mapped to the vertical by using a single layer approximation for the ionosphere at h_{sp}=400 km height. Using the GPS ground stations of the European IGS network, about 60-100 TEC data points are available for reconstructing TEC maps over the area 20°W ≤ λ ≤ 40°E, 32.5°N ≤ φ ≤ 70°N. To ensure a high reliability of the TEC maps the measurements are combined with the NTCM2 empirical model [5]. For each grid point value (spacing 2.5°/5° in latitude/longitude) a weighting process between nearest measured values and model values is carried out. The achieved accuracy for TEC is in order of $2\text{-}3\times10^{16}m^{-2}$ [6]. A linear interpolation algorithm within the corresponding grid pixel is applied to produce a TEC value at an arbitrary location. Interpolations at ionospheric station sites are important for obtaining the TEC dependence on geomagnetic activity and for density reconstruction and evaluation [7].

The ionosphere-plasmasphere system existence and variability is mostly determined by the solar and geomagnetic activity. A proper choice of indices is required for the TEC forecasting purposes.

2.2 Solar activity

The index chosen to represent the solar activity is F10.7. To better analyse the annual behaviour, the TEC 31-day running median values are normalised to the linear approximation of the TEC variations for the current year. The TEC response to the solar activity is rather complex (Fig.1). First, the TEC is strongly bound to the solar activity and its variability is significantly increasing with rising solar activity. Second, the summer maximum observed at low solar activity (LSA) is eroded at high solar activity (HSA), while two equinox maxima are appear to strongly dominate the annual profile. During day-time, the summer peak, observed at LSA, is gradually disappearing at higher solar activity. Night-time, the same happens to the LSA winter peak, which also gives way to two peaks observed near the HSA equinoxes.

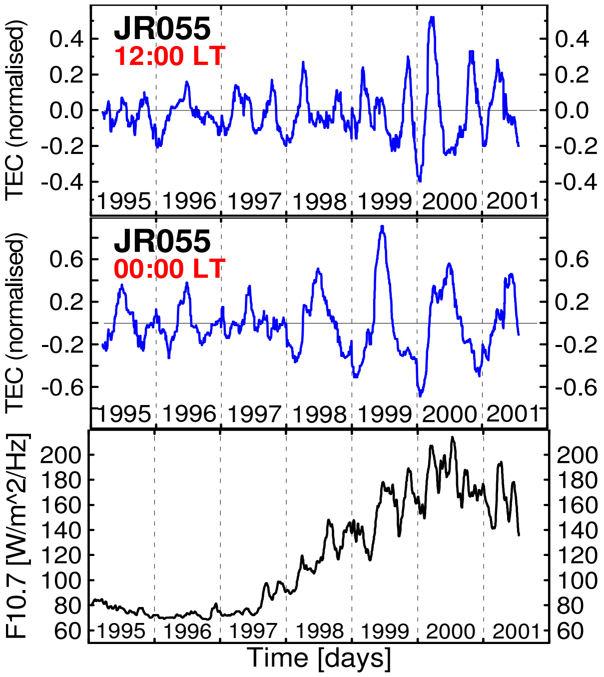

Fig.1 Normalised TEC 31-day running medians, JR055 site.

2.3 <u>Geomagnetic activity</u>

Many indices have been developed during the years for expressing the geomagnetic activity: the Kp and Ap planetary indices, the storm Dst index, the sub-storm index PC, etc. The Kp and Ap indices are probably the most suitable for reference when carrying out preliminary correlative studies with other related geophysical phenomena. A three-day forecast of the Ap index is issued by the NOAA Space Environment Center in Boulder USA, equipping the TEC forecasting method with one of the key input parameters. The daily Ap is nominally assigned to the 12:00LT hour and the hourly Ap values (denoted Aph) are obtained by linear interpolation. The Aph values are then converted to Kph through the established empirical relation Kp=1.739ln(0.423Ap).

The relative deviation, $F_r = (F-F_{med})/F_{med}$, of a given ionospheric parameter F (e.g. foF2, M3000F2, or TEC) from its median value, F_{med} , depends on the Kp index in a non-linear fashion. A similar study [3] on foF2 shows that the average dependence of the foF2 relative deviations on the Kp index has a parabolic form. However, the analysis of the TEC data reveals that, while the dependence of the relative TEC on Kph is similar to that of foF2 under quiet conditions (Kph<4), the dependence under disturbed conditions is quite complex (Fig.2). Of course, the TEC time series is not as long as that of foF2, and the data scattering is rather large, so further analysis is required. To linearise the dependence, a new 'synthetic' index is utilised, which is a polynomial approximation of the TEC average relative deviation depending on the hourly index Kph. It has been proved that the cross-correlation between the relative deviation of the critical frequency and the geomagnetic index Kp is much higher when using the squared Kp (i.e. Kp^2) than when using Kp itself [3]. Also, the ionosphere reaction to the geomagnetic forcing is delayed [4], so it is reasonable to describe this reaction as a dynamic relaxation process (i.e. exponentially decreasing magnitude) developing from a given initial stage. Thus, instead of using the Kp index, another index Km is introduced (named modified Kp) which is defined as a solution of the following ordinary differential equation of first order:

$$T\frac{dK_m(t)}{dt}+K_m(t)=K_p^2(t)$$

The left-hand side of this equation describes the relaxation of Km with a time constant T, while the term on the right-hand side represents the perturbation imposed on the pure relaxation of Km. Practically, the time constant T is a measure of the delayed reaction of the ionosphere to the geomagnetically-induced perturbations, and it is found that T is highly variable (T≈18h). By adjusting the time constant, the Km function can be obtained very close to the function describing the relative deviation of a parameter.

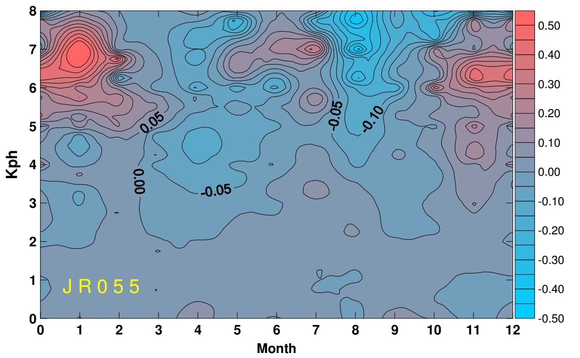

Fig.2 The TEC-based synthetic geomagnetic index obtained for station JR055 (15/02/1995 - 31/07/2001).

3. FORECASTING METHOD

The main idea in the proposed forecasting method is to consider the TEC temporal behaviour as composed of a periodic component and a random component. The periodic component represents the average (annual, diurnal) non-disturbed variation (traditionally represented by monthly medians), while the random component represents the perturbations inflicted on the TEC behaviour due to the strong changes in solar/geomagnetic activity. The method consists of two major components: (i) Extrapolation of the TEC monthly median values for up to 15 days ahead. The procedure uses Fourier series approximation based on actual data from the past 12 months and autocorrelation adjustment over the past 30 days of data. (ii) Forecast of the relative deviations of the measured TEC from its median values for up to 24 hours ahead. This forecast depends on the Kp index and is adjusted to the current conditions through an autocorrelation procedure.

3.1 Monthly median forecast

The monthly-median extrapolation procedure uses Fourier series approximation based on actual data from the past 12 months. The reasons we base the forecast on the monthly median are: i) the smooth annual variability of the TEC median value for a given hour, and ii) the similar pattern exhibited by the TEC measurements at all stations.

The TEC median value for each hour of the day is separately approximated using the following Fourier series decomposition:

$$TEC(d;h) = \frac{1}{2}a_0(h) + \sum_{i=1}^{n_{\max}}\left[a_i(h)\cos\left(i\frac{2\pi}{360}d\right) + b_i(h)\sin\left(i\frac{2\pi}{360}d\right)\right]$$

where d is the day of the year, h is the local time, and n_{max} is the number of the harmonic with the highest frequency. The procedure can be used for both interpolation and prediction purposes. If a one-day long gap is placed at the end of the data period, then the method will implicitly extrapolate the values within the imposed gap and will thus offer prediction values up to the length of the gap. In this case less data is required (30 days) to make a reliable one-day prediction. It is justified because the diurnal variations are much more stable than the annual. Both types of extrapolations (annual and diurnal) are used here and the resulting prediction is the average of both values for a given hour.

3.2 Adjustment of the monthly median forecast

The monthly-median extrapolation procedure is improved by an auto-correlation adjustment based on data from the last 30 days. This procedure relies on the diurnal rather than the annual behaviour of TEC. In the presented adjustment procedure, instead of the parameter's absolute value, the relative deviation from its median is used, F_r = (TEC-TEC$_{\text{med}}$)/TEC$_{\text{med}}$. This function contains no periodic components and can be regarded as a steady-state random process over the considered period of time. Having measurements of F_r at times t_i (i=1,2,...,n), the aim is to predict the values of F at future moments t_{n+1}, t_{n+2}, ..., t_{n+k} where (t_n,t_{n+k}) is the length of the prediction period. For the purpose, a regression model [1] is used :

$$F(t_{n+1}) = F_m + \sum_{i=1}^{n}\beta_i\left(F(t_i) - F_m\right)$$

where F_m is the sample's median. The weight coefficients are determined from the following system:

$$\sum_{i=1}^{n}\beta_i r_{FF}(t_i - t_j) = r_{FF}(t_{n+1} - t_j)\ ,\ j = 1,2,...,n$$

where r_{FF} is the auto-correlation function of F. Practically, the real r_{FF} is unknown, so the normalised empirical auto-correlation function is used instead:

$$\rho_{FF}(\tau) = \frac{\sum F_i F_{j(i)}}{\sqrt{\left(\sum F_i^2\right)\left(\sum F_{j(i)}^2\right)}}$$

where the summation is performed over the pairs of *F(t)* having a same time difference τ. It is implicitly assumed that the empirical mean is zero, which is justified for series of several days. Also, because the empirical auto-correlation function is only an estimate of the true auto-correlation function, the accuracy of that estimate depends on the size of the data sample, which should be much larger than the number of the unknown coefficients.

3.3 Short-term prediction

The TEC short-term forecast is improved by introducing cross-correlation between the TEC relative deviation from its median value and the modified index Km. Thus, the following regression formula [3] is used for predicting the relative deviation:

$$F_{n+1} = F_m + \sum_{i=1}^{n}\beta_i\left(F_i - F_m\right) + \sum_{i=1}^{n+1}\beta_i\left(G_i - G_m\right)$$

where *G(t)* is the geomagnetic function (approximation of the modified geomagnetic index Km). The weight coefficients are determined from the following system:

$$\sum_{i=1}^{n}\beta_i\rho_{FF}(\tau_{ij}) + \frac{\sigma_G}{\sigma_F}\sum_{i=1}^{n+1}\gamma_i\rho_{FG}(\tau_{ij}) = \rho_{FF}(\tau_{n+1,j})$$

$j = 1\ ,\ ...\ ,\ n$

$$\sum_{i=1}^{n}\beta_i\rho_{GF}(\tau_{ij}) + \frac{\sigma_G}{\sigma_F}\sum_{i=1}^{n+1}\gamma_i\rho_{GG}(\tau_{ij}) = \rho_{GF}(\tau_{n+1,j})$$

$j = 1\ ,\ ...\ ,\ n+1$

The auto-correlation functions depend on the time shift, $\tau_{ij}=t_i-t_j$, only. Here, ρ_{FG} (note that $\rho_{\text{FG}}=\rho_{\text{GF}}$) is the cross-correlation function between *F(t)* and *G(t)*, defined

$$\rho_{FG}(\tau) = \frac{\sum F_i G_{j(i)}}{\sqrt{\left(\sum F_i^2\right)\left(\sum G_{j(i)}^2\right)}}$$

where again the summation is performed over the pairs of *F(t)* having the same time difference τ in the data sample. The *F* and *G* auto-correlation functions are ρ_{FF} and ρ_{GG}, and the corresponding standard deviations are :

$$\sigma_F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}F(t_i) - \left(\frac{1}{n}\sum_{i=1}^{n}F(t_i)\right)^2}$$

$$\sigma_G = \sqrt{\frac{1}{n+1}\sum_{i=1}^{n+1}G(t_i) - \left(\frac{1}{n+1}\sum_{i=1}^{n+1}G(t_i)\right)^2}$$

Statistical sufficiency of the auto-correlation functions is ensured by using parametric expressions. The auto-correlation function of *G* decreases exponentially with increasing the time difference, τ , which leads us to the conclusion that this auto-correlation function has the following form:

$$R_{GG}(\tau) = G_{mean} + \sigma_G^2\exp\left(-|\tau|/T_G\right)$$

known in the signal-processing theory as 'random telegraphic wave'. The auto-correlation is symmetrical with respect to τ=0 and decreases with a time constant T_G. In resume, the geomagnetically-correlated forecasting procedure works as follows. First, the TEC data, from the 30-day period prior to the date of forecast, are assembled and the TEC hourly medians are calculated. Second, all TEC relative deviations from the median value are computed. Third, the Kph index is obtained over the whole data period including the day of the prediction. Next, all values G_i of the geomagnetic function are derived. Further, based on the available data from the 30-day period, the cross- and auto-correlation functions of F and G are determined together with the weighting coefficients. Finally, the required 24 values of F_r for the forecasted day are obtained using the regression formula.

4. RESULTS AND DISCUSSION

The TEC hourly time series is considered as a sum of two components - periodic and random. The periodic component is non-random and describes the TEC mean behaviour (represented here by the 31-day running medians). On the other hand, the random component describes the TEC fluctuations supposedly inflicted by the geomagnetic field disturbances. These fluctuations are supposed to be a manifestation of a stationary stochastic process. The stationarity hypothesis implies that the mean E{n(t)} and the product moment E{n(t)n(t+τ)} are independent on t , where E denotes the mathematical expectation. Such interpretation suggests that the TEC median behaviour is the signal, and the fluctuations are noise. The method has been tested for 24-hour median predictions; exemplary test results are provided in Fig.3 for high solar activity. The first prediction hour is 00:00 LT and the last 23:00 LT of the same day. In the top panel, the averaged values of the diurnal and annual extrapolation is given. In the bottom

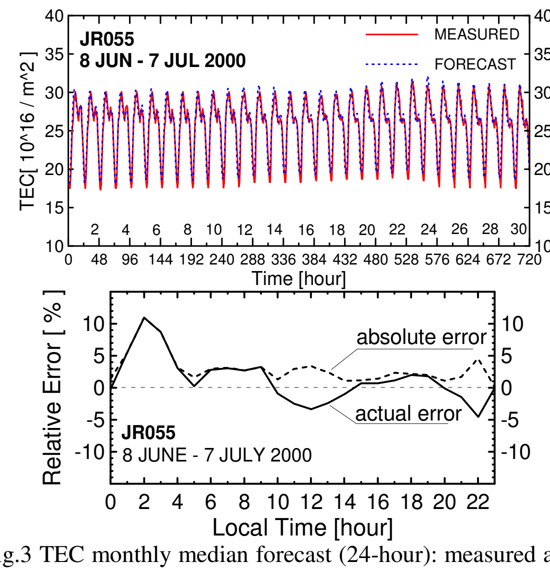

Fig.3 TEC monthly median forecast (24-hour): measured and forecast values (top) and relative errors (bottom), JR055 site.

panel, the absolute and real values of the relative error (averaged within each hour of the day) are plotted. The relative errors are larger during night (reaching occasionally 10%) but in the rest of the time are varying between 1 and 3%. The results of the geomagnetically-correlated short-term prediction of ionospheric parameters are presented and discussed elsewhere [3] and will not be repeated here.

Basic sources of errors in this forecasting method are: the insufficiency of data measurements within a certain area, inevitable data smoothing inherent in the mathematical formulation, unknown and unstable relationship between the ionospheric characteristic and the solar-geomagnetic activity, etc.

5. CONCLUSIONS

A new method for monthly median and short-term prediction of TEC has been presented. First, the TEC monthly medians are extrapolated using Fourier series approximation based on actual data from the past 12 months and autocorrelation adjustment over the past 30 days of data. Second, the TEC relative deviation from its median is extrapolated for up to 24 hours ahead using geomagnetically-correlated regression model. The advantages of the presented approach are in the using of routine reliable measurements, employing established numerical methods, offering real-time forecasting capabilities, etc. Therefore, this TEC short-term forecasting can be a powerful instrument for both research and operating practice and might be considered in many space-weather related applications.

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REFERENCES

1. Muhtarov P., Kutiev I., Autocorrelation method for temporal interpolation and short-term prediction of ionospheric data, *Radio Sci.*, **34**, 459-464, 1999.
2. Kutiev I., Muhtarov P., Cander L., Levi M., Short-term prediction of ionospheric parameters based on auto-correlation analysis, *Ann.Geofisica*, **42**, 121-127, 1999.
3. Muhtarov P., Kutiev I., Cander L., Geomagnetically correlated autoregression model for short-term prediction of ionospheric parameters, *Inv.Problems Physics*, **17**, 1-17, 2001.
4. Kutiev I., Muhtarov P., Modelling the midlatitude response to geomagnetic activity, *J.Geoph.R.*, **106**, 15501-15510, 2001.
5. Jakowski N., TEC Monitoring by Using Satellite Positioning Systems, *Modern Ionospheric Science, (Eds. H.Kohl, R.Rüster, K.Schlegel)*, EGS, 371-390, 1996.
6. Jakowski N., Sardon E., Engler E., Jungstand A., Klaehn D., Relationships between GPS-signal propagation errors and EISCAT observations, *Ann.Geophys.*, **14**, 1429-1436, 1996.
7. Stankov S.M, Kutiev I., Jakowski N., Heise S., Two-slope electron density profiles based on GPS TEC, O^+-H^+ transition height and ionosonde data, *COST271 Workshop, Sopron,* 2001.