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THE DETERMINATION OF LAY-PARAMETERS FOR A GIVEN PROFILE

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ABSTRACT

An algorithm is presented by which the function LAY can be inverted so that the geometric parameters HX and SC of such function used to describe a given profile in the neighborhood of a peak can easily be determined.

DESCRIPTION OF ALGORITHM

In the development of the IRI-programme, it may be interesting to consider the representation of a given profile in the neighborhood of an extremum. For this we use a particular EPSTEIN-function : the LAY-function defined as /1/

$$LAY(xm;x) = A\left[\mathrm{Ln}\left(\frac{1+e^{x}}{1+e^{xm}}\right) - (x - xm)\frac{e^{xm}}{1 + e^{xm}}\right] \qquad (1)$$

where $x = (z - HX)/SC \quad , \quad xm = (zm - HX)/SC \qquad (2)$

with : z the height coordinate, zm the height of the extremum, HX a height parameter and SC a scale parameter.

We restrict our discussion to LAY-functions with the amplitude factor $A = \pm 1$ so that the functions have positive or negative values depending on the $\pm$ sign.

We consider a height interval over which a given profile of the electron concentration has a curvature of a certain sense. In this height range we select three altitudes : z1, z2 and zm the altitude of the extremum. Then we look for the LAY-function fitting with the profile at these three levels according to the conditions

$$LAY(xm;xm) = 0 \ , \ LAY(xm;xi) = \log_{10}(N_{ext}/N(zi)) \quad (i = 1,2)$$

An algorithm has been found which in an iterative way rapidly determines the parameters HX and SC. It uses an analytical approximation, which for given values of LAY(xm;x) and of xm, allows to calculate x with fair accuracy.

Writing Y for LAY(xm;x) the inversion $x = LAY^{-1}(xm;Y)$ may be written as

$$x = LAY^{-1}(xm;Y) = A(Y) + B(Y) \exp - \left[\sum_{j=1}^{6} C_j(Y)\,(xm)^j\right] \qquad (3)$$

with $A(Y) = -\mathrm{Ln}(e^{Y}-1), \quad B(Y) = 2\,\mathrm{Ln}\left[e^{Y} - (e^{2Y} - 1)^{\frac{1}{2}}\right]$

where in fact : $A(Y) = LAY^{-1}(\infty;Y)$ describes the asymptotic behavior of x
$B(Y) = LAY^{-1}(0;Y) - A(Y)$ depends on the particular case xm = 0.

As for the coefficients $C_j(Y)$ of the polynomial argument of the exponential these are obtained by interpolation from a table which was computed by the author and is available from him.

In order to compute HX and SC when applying this algorithm to a given profile we note that

$$R = \frac{zm - z2}{zm - z1} = \frac{xm - x2}{xm - x1}$$

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is independent of the actual (and unknown) values of HX and SC. Furthermore, applying the inversion equation (3), one sees that R is a function of xm only. Thus for the three points chosen on the given profile the corresponding values of xm, x1 and x2 can be found by a trial and error procedure.

Once xm and x1 or x2 are known, equations (2) give the corresponding values of HX and SC.

APPLICATIONS

In order to check the utility of this algorithm we have considered profiles of the electron concentration in the middle ionosphere, particularly in the F2-region, as given by IRI79 /2/ for the locations Boston and Dakar.

The main results of the computations are :
a) the fit is practically valid in the whole F2-region (Figure 1)
b) the ratio HX/zm and the parameter SC exhibit consistent variations with time, season and solar activity (Figure 2).

This algorithm might also be used to establish a "climatological data base" of HX and SC values by online processing of measured real height profiles.

This can most easily be achieved when using the real heights corresponding to given electron densities or better for frequencies expressed as a fixed ratio of the peak frequency. More precisely, starting with z(fo) = zm, z1 = z(k1.fo) and z2 = z(k2.fo), a very short computational effort is needed to go from R = (zm - z2)/(zm - z1) to xm and x1 and finally to HX and SC.

The same computation may be repeated for different pairs of ratios k1 and k2 leading to new pairs of values of HX and SC. From this dataset, after removing of aberrant cases, one can determine a representative pair HX and SC.

When profiles are computed from ionograms using a program like POLAN /3/ or the method described in /4/ the proposed determination of HX and SC may easily (and without much additional computing time) be implemented into the main program.

ACKNOWLEDGMENT

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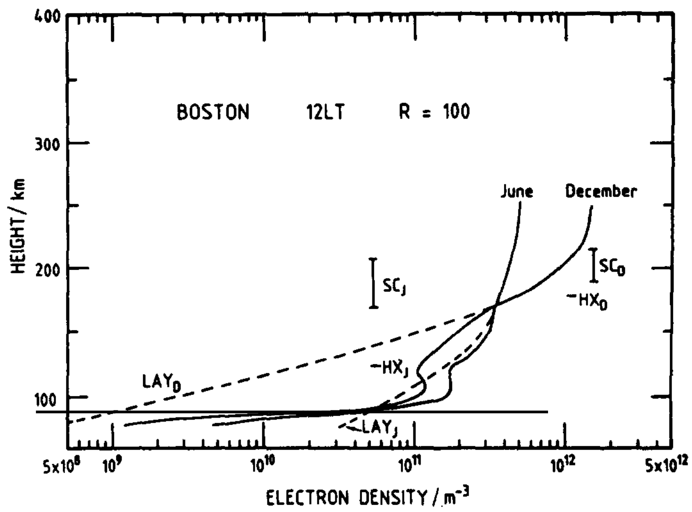

Fig. 1. Examples of fitting. The plain curves are the given profiles, the dashed which coincide with the plain in the F2-region are the LAY-functions. The heights HX are indicated and the bars give the scale parameters SC. The labels J and D indicate the month.

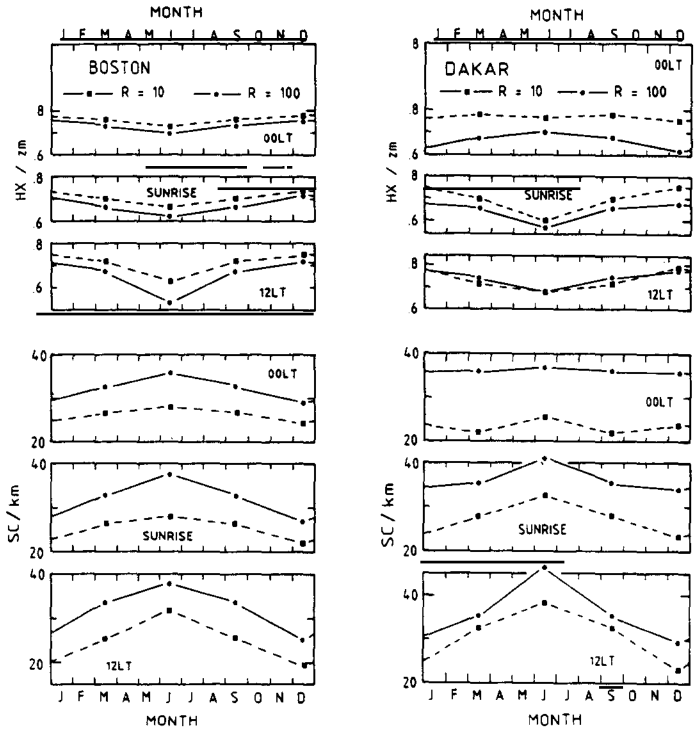

Fig. 2. Variation of HX/zm and SC determined from the profiles of IRI79 for Boston and Dakar.