THE DETERMINATION OF LAY-PARAMETERS FOR A GIVEN PROFILE

L. Bossy *

Institut d'Astronomie et de Géophysique G. Lemaître, Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium

ABSTRACT

An algorithm is presented by which the function LAY can be inverted so that the geometric parameters HX and SC of such function used to describe a given profile in the neighborhood of a peak can easily be determined.

DESCRIPTION OF ALGORITHM

In the development of the IRI-programme, it may be interesting to consider the representation of a given profile in the neighborhood of an extremum. For this we use a particular EPSTEIN-function: the LAY-function defined as /1/

\[
\text{LAY}(x_m;x) = A \ln \left( \frac{1 + e^{x - x_m}}{1 + e^{-x + x_m}} \right)
\]

where \( x = (z - HX)/SC, \ x_m = (zm - HX)/SC \) (2)

with \( z \) the height coordinate, \( zm \) the height of the extremum, \( HX \) a height parameter and \( SC \) a scale parameter.

We restrict our discussion to LAY-functions with the amplitude factor \( A = \pm 1 \) so that the functions have positive or negative values depending on the sign.

We consider a height interval over which a given profile of the electron concentration has a curvature of a certain sense. In this height range we select three altitudes: \( z_1, z_2, zm \) the altitude of the extremum. Then we look for the LAY-function fitting with the profile at these three levels according to the conditions

\[
\text{LAY}(x_m;xi) = \frac{\log_{10}(N_{ext}/N(z_i))}{(i = 1, 2)}
\]

An algorithm has been found which in an iterative way rapidly determines the parameters HX and SC. It uses an analytical approximation, which for given values of LAY(xm;x) and of \( x_m \), allows to calculate \( x \) with fair accuracy.

Writing Y for LAY(xm;x) the inversion \( x = \text{LAY}^{-1}(x_m;Y) \) may be written as

\[
x = \text{LAY}^{-1}(x_m;Y) = A(Y) + B(Y) \exp \left[ \sum_{j=1}^{6} C_j(Y) (x_m)^j \right]
\]

with \( A(Y) = - \ln(y^2 - 1), \ B(Y) = 2 \ln \left[ y - (y^2 - 1)^{\frac{1}{2}} \right] \)

where in fact \( A(Y) = \text{LAY}^{-1}(x_m;Y) \) describes the asymptotic behavior of \( x \)

\( B(Y) = \text{LAY}^{-1}(0;Y) = A(Y) \) depends on the particular case \( x_m = 0 \).

As for the coefficients \( C_j(Y) \) of the polynomial argument of the exponential these are obtained by interpolation from a table which was computed by the author and is available from him.

In order to compute HX and SC when applying this algorithm to a given profile we note that

\[
R = \frac{zm - z_1}{zm - z_2} = \frac{zm - x_1}{zm - x_2}
\]

* also Institut d'Aéronomie Spatiale de Belgique, 3, Avenue Circulaire, B-1180 Bruxelles, Belgium.
is independent of the actual (and unknown) values of \( H_X \) and \( SC \). Furthermore, applying the inversion equation (3), one sees that \( R \) is a function of \( x_m \) only. Thus for the three points chosen on the given profile the corresponding values of \( x_m, x_1 \) and \( x_2 \) can be found by a trial and error procedure.

Once \( x_m \) and \( x_1 \) or \( x_2 \) are known, equations (2) give the corresponding values of \( H_X \) and \( SC \).

APPLICATIONS

In order to check the utility of this algorithm we have considered profiles of the electron concentration in the middle ionosphere, particularly in the F2-region, as given by IRI79 /2/ for the locations Boston and Dakar.

The main results of the computations are:

a) the fit is practically valid in the whole F2-region (Figure 1)

b) the ratio \( H_X/x_m \) and the parameter \( SC \) exhibit consistent variations with time, season and solar activity (Figure 2).

This algorithm might also be used to establish a "climatological data base" of \( H_X \) and \( SC \) values by online processing of measured real height profiles.

This can most easily be achieved when using the real heights corresponding to given electron densities or better for frequencies expressed as a fixed ratio of the peak frequency. More precisely, starting with \( z(fo) = x_m, z_1 = z(k_1.fo) \) and \( z_2 = z(k_2.fo) \), a very short computational effort is needed to go from \( R = (z_m - z_2)/(z_m - z_1) \) to \( x_m \) and \( x_1 \) and finally to \( H_X \) and \( SC \).

The same computation may be repeated for different pairs of ratios \( k_1 \) and \( k_2 \) leading to new pairs of values of \( H_X \) and \( SC \). From this dataset, after removing of aberrant cases, one can determine a representative pair \( H_X \) and \( SC \).

When profiles are computed from ionograms using a program like POLAN /3/ or the method described in /4/ the proposed determination of \( H_X \) and \( SC \) may easily (and without much additional computing time) be implemented into the main program.

ACKNOWLEDGMENT

This work has been supported in part by Fonds National de la Recherche Scientifique - Crédits au Chercheurs.

REFERENCES


Fig. 1. Examples of fitting. The plain curves are the given profiles, the dashed which coincide with the plain in the F2-region are the LAY-functions. The heights $H_X$ are indicated and the bars give the scale parameters $S_C$. The labels J and D indicate the month.

Fig. 2. Variation of $H_X/\chi_{\infty}$ and $S_C$ determined from the profiles of IRI79 for Boston and Dakar.