

## HIGH-LATITUDE IONOSPHERIC MODEL

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### ABSTRACT

A set of regional maps of peak electron density in the northern polar cap was deduced from aeronomical computations for southward IMF executed at Utah State University (USU) /1/. Each of the 12 maps holds for invariant latitudes greater than 50° N, covering 24h both in magnetic local time and in UT. The maps for two parameters are analyzed in terms of Empirical Orthogonal Functions (EOF). The dependence on other parameters is described by continuous functions. In the northern polar cap, these maps might be used as a partially substituting supplement of the world wide maps recommended by the CCIR/2/.

### INTRODUCTION

The Utah State University (USU) database is a set of numerical tables calculated using aeronomical theory /1/. The 'case studies' depend on a few empirical inputs, in particular, the total corpuscular energy input /3/ and the distribution of horizontal electric fields /4/. These two factors cause the high latitude ionosphere to differ fundamentally from that at lower latitudes. In order to supplement the ionization maps of the CCIR /2/ at high northern latitudes, we have analyzed the USU database by Dvinskikh's method of Empirical Orthogonal Eigenfunctions. This permits adequate smoothing and a considerable reduction in the data that must be stored /5/.

### MATRIX REPRESENTATION AND SMOOTHING

The original data charts consist of matrices ( $N_e$ ) of 20 x 24 data points giving the peak electron density as function of invariant latitude (MLAT) and magnetic local time (MLT). Following /6/, we first construct a symmetric matrix

$$D = N_e * N_e^T \quad (1)$$

using the transposed matrix  $N_e^T$ .

The eigenvalues and the associated eigenvectors of  $D$  provide an eigenvector matrix  $U$ , with the eigenvectors as columns in the order of decreasing eigenvalues. Defining a matrix  $V$  by

$$V = U^T * N_e \quad (2)$$

the original matrix can be written as

$$N_e = U * V \quad (3)$$

The smaller eigenvalues are ignored when smoothing, i.e., only the  $k$  first columns in  $U$  and the first  $k$  lines in  $V$  are retained, giving

$$N_{ek} = U_k * V_k \quad (4)$$

The difference between square sum of  $N_e$  and  $N_{ek}$  equals the sum of the eigenvalues of the omitted eigenvectors.

Figure 1 shows the smoothing effect of different values  $k$  ( $k = 2, 6, 14$ ) relative to the original map. Reasonable smoothing with good reproduction of significant features is obtained for  $k = 6$ . The standard deviation from the original matrix is about 1%, while the storage capacity needed is reduced from 480 to 264.

### UNIVERSAL TIME DEPENDENCE

For prescribed season, magnetic activity and solar level, 12 eigenvalue matrices  $U$  and Dvinskikh matrices are found as a function of UT. Every matrix element of  $U$  and  $V$  is Fourier-analyzed in universal time. The original data occasionally show significant noise, which is minimized by taking the median values for the UT-profiles. Figure 2 shows the Fourier approximations using 13 (original), 7, 5 and 3 coefficients. The original data can be represented with sufficient accuracy by five Fourier-coefficients. The five Fourier-coefficients for all elements of

the eigenvector- and the Dvinskikh-matrix describe five matrices, which define the UT-dependence. The original data set with  $480 * 12 = 5760$  values is now reduced to 1320 values (23%).

#### MAGNETIC LOCAL TIME DEPENDENCE

The variation in MLT is shown by the Dvinskikh definition matrix  $V$  or its UT Fourier-analyzed version FCV. The Fourier analysis of the corresponding matrix adequately reproduces the periodic features. For one selected data set (UT = 1, medium magnetic activity, high solar activity, winter) the approximation is extremely good for the first four matrix elements (Figure 3), even with only five Fourier coefficients. The last two orders are not so well represented but have little numerical importance. Calculating  $N_m F_2$ -maps with different numbers of Fourier coefficients the gross features of this particular map were well represented with only five coefficients. The number of original data points is reduced from 5760 points to finally 750 numbers (13%).

#### DEPENDENCE ON MAGNETIC ACTIVITY AND SOLAR LEVEL

No straightforward procedure to introduce the magnetic activity and solar level dependencies could be found. Difference maps, obtained by subtracting, e.g., the map for  $K_p = 6$  from that for  $K_p = 3.5$  (Figure 4) show the complexity.

Complex MLAT-MLT patterns were found for changes in the solar level or season. We, therefore, adopt a binned  $K_p$  treatment for medium and high magnetic activity and a binned  $F_{10.7}$  treatment for low and high solar level. For intermediate  $K_p$  and  $F_{10.7}$  values, the  $N_m F_2$  density is found by linear interpolation. Seasonal effects are included using winter, equinox and summer descriptions. The initial amount of data is thus finally reduced from 69,120 to 9,000 data points or 13%.

#### APPLICATION

While the USU data set is tabulated in the MLAT-MLT frame, the low- and mid-latitude model /7/ actually applied by ESOC uses geodetic coordinates as needed in applications.

From given geographic latitude and longitude we calculate magnetic local time, MLT, after /8/. McIlwain's  $L$ -shell parameter for a representative altitude of 300 km is derived from the international geophysical reference field IGRF from 1945 to 1985, extrapolated to the period 1985 to 1990 /9/. The shell parameter  $L$  finally gives the invariant latitude /10/.

These coordinate transformations give the polar  $N_m F_2$ -values in the original frame. Therefore, we first compose the Dvinskikh matrix by two Fourier syntheses in MLT and UT and the eigenvector matrix by one Fourier synthesis in UT. The approximated model density values are derived from the product of  $U$  and  $V$  according to equation (4).

The original and the approximated map for UT = 1 are compared in Figure 5. The average error is below 4%. The occasional errors greater than 10% are due to smoothing (by taking the median values in the UT-variation of the Fourier-components). Such isolated peaks occasionally observed in the original data, are always eliminated for prediction purposes. The model is available from NASA WDC-A, R&S in Greenbelt, MD. 20771, USA (also from SPAN).

#### REFERENCES

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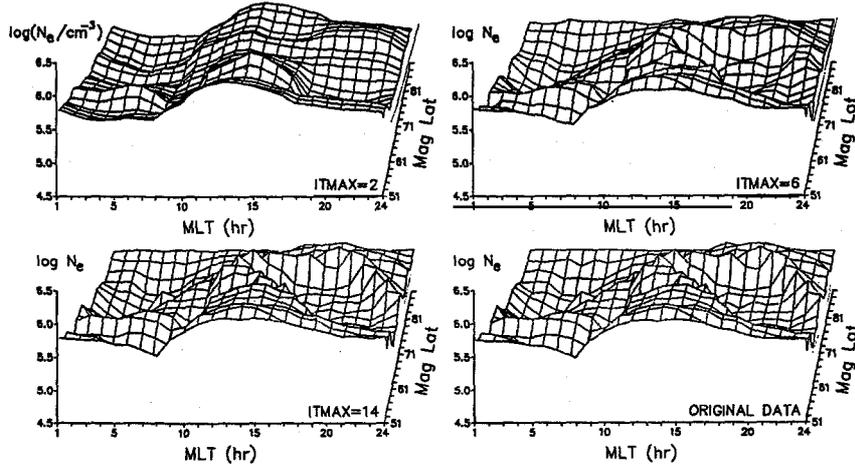


Fig. 1. Approximated electron density maps for different iteration cut-offs ITMAX compared to the original data.

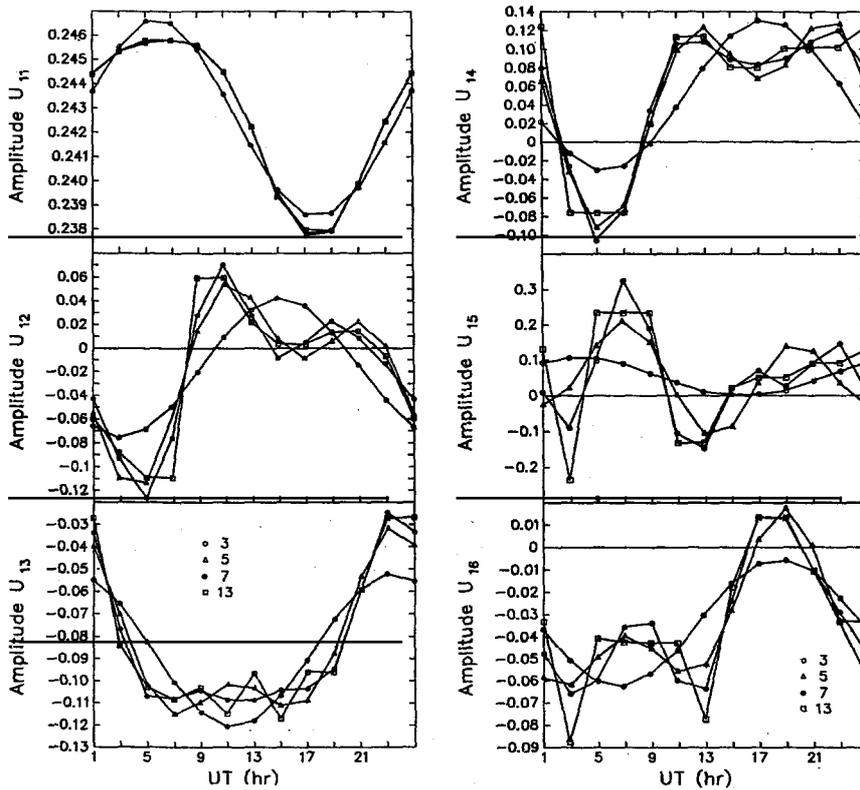


Fig. 2. Different approximations for the six components of the first eigenvector using 13, 7, 5 and 3 Fourier-coefficients. The original noise of the highest order terms is eliminated by using median values.

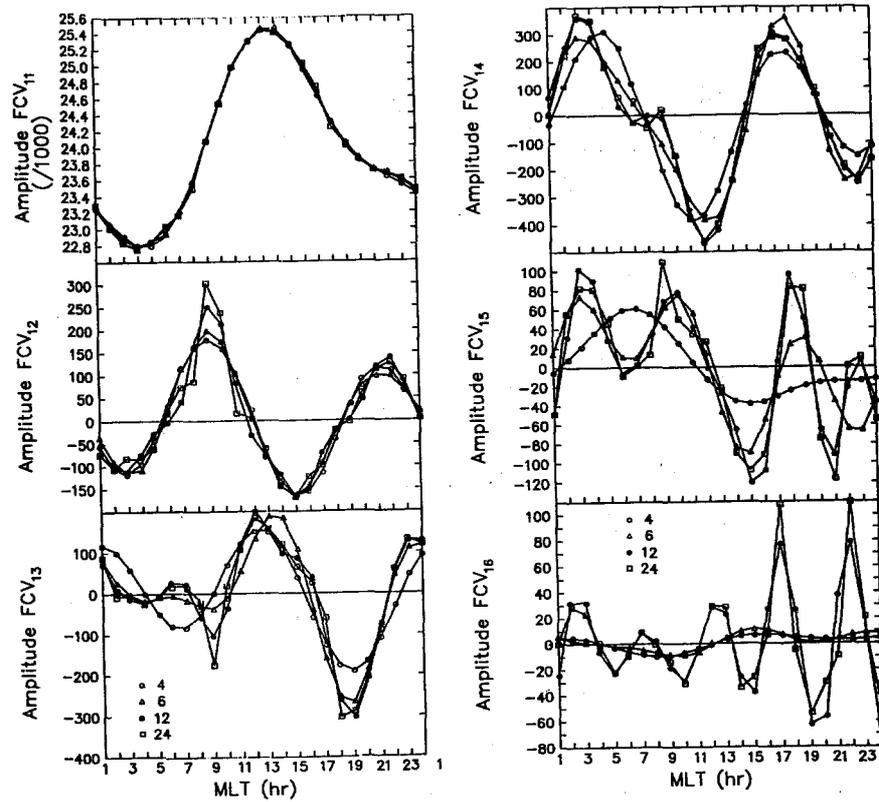


Fig. 3. Approximation for UT-Fourier matrix using the indicated number of MLT-Fourier coefficients. Input parameters are UT = 1, medium magnetic activity, high solar level and winter.

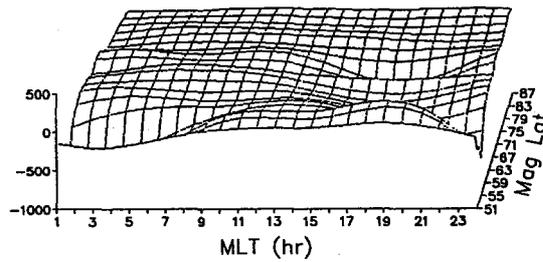


Fig. 4. Difference density map, summer season and high solar level, for  $K_p = 3.5$  and  $K_p = 6$ .

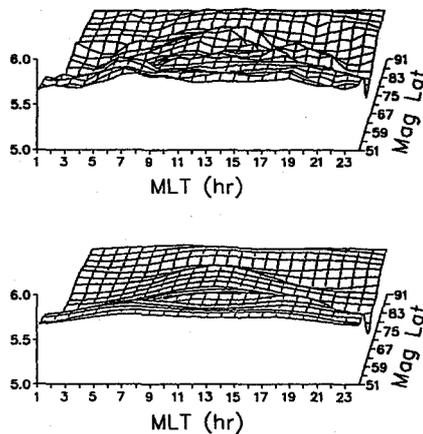


Fig. 5. Original and model density map for UT = 3, medium magnetic activity, high solar level and winter.