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Effects of the atmospheric scale height gradient on the variation of ionization and short wave absorption

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ABSTRACT

A discussion of the effect of a variable atmospheric scale height is given. A linear gradient of the scale height is used and it is found that special conditions for ionized layer formation are possible. Also, the variation of the layer critical frequency with the altitude of the sun above the horizon and of the short wave absorption are found to be affected by the variation of the scale height. The scale height variation is involved in expressions for the absorption of the solar radiation and the electronic recombination coefficient.

I

The process of ionized layer formation has been examined in detail by CHAPMAN [1]. The simple case of ionization may be considered by assuming the absorption of ultra-violet light with a definite absorption coefficient in an atmosphere of uniform composition and temperature. Furthermore, the recombination coefficient is generally supposed constant independent of height. This direct method yields information regarding the possible origins of the ionospheric layers, the vertical distribution of the electronic concentration and the variation of the critical frequency with the solar zenith distance.

The origin of the ionospheric layers or the determination of the photo-ionization processes may be studied by way of the following formula

$$NKH = \cos X \tag{1}$$

N being the active constituent's concentration at the maximum of absorption, K being the molecular absorption coefficient associated with the photo-ionization process, H being the local scale height and X being the solar zenith distance. The value of the NKH product indicates the level of the layer if K is known.

The distribution of the electronic concentration N_e with height is expressed (in equilibrium) by

$$N_e = N^*_{eM} e^{\frac{1}{2}(1 - \zeta - e^{-\zeta} \sec X)} \tag{2}$$

where N^*_{eM} is the maximum electronic concentration for $X = 0°$ (overhead sun) and $\zeta = (z - z^*_M)/H$, z being the height above a certain level $z = 0$, z^*_M being the value of z corresponding to N^*_{eM}.

The theoretical variation of the critical frequency f_c with the solar zenith distance is represented by

$$f_c = f_c(X = 0°) \times (\cos X)^{\frac{1}{4}} \tag{3}$$

An important result has been obtained by APPLETON [2] regarding the total absorption suffered by a short radio wave when it travels a non-deviating CHAPMAN region. For the ordinary wave the absorption coefficient A is given by

$$A = \frac{2\pi e^2}{mc} \; \frac{N_e \nu}{\nu^2 + (p + |p_L|)^2} \tag{4}$$

where ν is the collisional frequency of the electrons of mass m and charge e, $f = 2\pi p$ is the exploring wave frequency, and $f_L = 2\pi p_L$ is the longitudinal gyromagnetic frequency.

If the absorption of short waves reflected from the F-region takes place in a non-deviating region, the reflection coefficient may be expressed simply as

$$-\log \rho \div (\cos \chi)^{3/2} \tag{5}$$

if we consider only the variation of the total absorption with the solar altitude.

Beyond this simple example, it is interesting to consider the possible observable effects on the characteristics of the ionosphere in cases where the structure of the atmosphere is more complicated and there is a variation of the electronic recombination coefficient with height. Nicolet and Bossy [3] have given some information regarding the effect of the scale height gradient on the law of variation of short wave absorption with solar zenith distance.

II

Let us first consider the atmospheric pressure p governed by the statical equation

$$dp = -gNM\,dz \tag{6}$$

where g denotes the acceleration of gravity and M the mean molecular mass. The pressure is also expressed by the equation of a perfect gas

$$p = NkT \tag{7}$$

where T denotes the absolute temperature and k Boltzmann's constant. Then (6) and (7) clearly give

$$\frac{dp}{p} = \frac{dN}{N} + \frac{dT}{T} = -\frac{dz}{H} \tag{8}$$

where the scale height follows a law of linear variation,

$$H = \frac{kT}{mg} = H_0 + \beta z \qquad (H = H_0 \text{ at } z = 0) \tag{9}$$

where

$$\beta = dH/dz = \text{constant} \tag{10}$$

If the molecular mass is considered as constant* (g is assumed as constant in a certain range of heights), equation (8) may be re-written in the equivalent form

$$\frac{dN}{N} = -\frac{1+\beta}{\beta}\frac{dH}{H} \tag{11}$$

With (9), the integration of (11) gives the height distribution of N

$$N = N_0 \,(H/H_0)^{-\frac{1+\beta}{\beta}} \tag{12}$$

The total number in a column between z and ∞ is then

$$\int_z^{\infty} N dz = NH \tag{13}$$

* It is not difficult to pursue the effect of a variation of the molecular mass ; but our purpose is only to give here some indications.

If we follow the details of CHAPMAN'S calculations, we find [3] expressions*

(a) $$N_M K H_M = (1 + \beta) \cos \chi \tag{14}$$

giving the maximum absorption law where N_M and H_M are the local values at the height of maximum absorption.

(b) $$N_e = N^*_{eM} \, e^{\frac{1+\beta}{2}\left(1 - \zeta - e^{-\zeta} \sec \chi\right)} \tag{15}$$

where ζ connects the local scale height H with the scale height at the absolute maximum H^*_M (overhead sun) by the relation

$$\left(H/H^*_M\right) = e^{\beta \zeta} \tag{16}$$

It is convenient to consider only the photo-ionization. In this case, instead of (15), we write the formula

$$q = q^*_M \, e^{(1+\beta)\left(1 - \zeta - e^{-\zeta} \sec \chi\right)} \tag{17}$$

where q^*_M is the "maximum number of photo-ionization" when $\chi = 0°$,

$$q^*_M = e \frac{(1+\beta)\, Q_\infty}{(1+\beta)\, H^*_M} \tag{18}$$

Q_∞ being the number of quanta arriving at the top of the atmosphere or of the considered layer.

Furthermore, the condition of a maximum $(dq/dz = 0)$ is

$$e^{-\zeta} = \cos \chi \tag{19}$$

and the maximum of photo-ionization q_M is

$$q_M = q^*_M \, e^{-(1+\beta)\zeta} \tag{20}$$

or, by (19),

$$q_M = q^*_M (\cos \chi)^{1+\beta} \tag{21}$$

which indicates the variation with solar zenith distance of the maximum number of photo-ionizations.

(c) If the recombination coefficient is constant with height, the critical frequency f_c is

$$f_c = f_c(\chi = 0°) \times (\cos \chi)^{\frac{1+\beta}{4}} \tag{22}$$

Under the same conditions, the absorption of short waves follows the law

$$-\log \rho \div (\cos \chi)^{\frac{3+\beta}{2}} \tag{23}$$

This shows how a variation of the scale height affects the behaviour of certain theoretical characteristics of the ionosphere. Nevertheless, the variation with height of the recombination coefficient cannot be neglected.

III

Taking $dN_e/dt = 0$, thus $N_e = (q/\alpha)^{\frac{1}{2}}$, we can determine the general condition of a maximum of the electronic concentration.

* The value of the absorption coefficient is a mean value.

Let τ be the optical depth at a height z, so that

$$d\tau = N_j\, K_j\, dz \sec \chi \tag{24}$$

$$\tau = N_j\, K_j\, H_j \sec \chi = K_j \sec \chi \int_z^{\infty} N_j\, dz \tag{25}$$

where N_j refers to the atom or molecule subject to the photo-ionization. We suppose the following relation with another constituent N_k

$$N_j = C\, N_k^n \tag{26}$$

where C will be considered as constant.

If Q_∞ is still the number of available quanta at the top of the layer we can write the ionization equilibrium equation

$$N_j\, K_j\, Q_\infty\, e^{-\int_z^{\infty} N_j K_j\, dz \sec \chi} = \alpha\, N_e^2 \tag{27}$$

α being the recombination coefficient.

Let us suppose α to be a function of N_k only, according to

$$\alpha = \alpha_o\, N_k^m \tag{28}$$

where α_0 = constant. Furthermore, we take $H_k = H_{k.o} + \beta z$ which determines the scale height so that N_k refers to the principal constituent and, [*cf* (11)]

$$\frac{dN_k}{N_k} = -\frac{1+\beta}{\beta}\frac{dH_k}{H_k} \tag{29}$$

Finally, with (26), (28), (29), the ionization equation (27) may be written as

$$C\, N_k^n\, K_j\, Q_\infty\, e^{-\frac{CN_k^n\, K_j\, H_k}{[n\,(1+\beta) - \beta] \cos \chi}} = \alpha_o N_k^m N_e^2 \tag{30}$$

Thus, the condition of a maximum of N_e is given by

$$\frac{N_j\, K_j\, H_k \sec \chi}{n\,(1+\beta)} = 1 - \frac{m}{n} \tag{31}$$

If the indices m and n (*cf* (26), (28)) are positive, the electronic concentration has a maximum only when $m < n$.

In the atmosphere, above the maximum of dissociation of O_2, n may be 2 or 3, according as the recombination to O_2 occurs by a two-body or three-body collision.

In the case of two-body radiative recombination, (31) becomes

$$N_j\, K_j\, H_k = (2 - m)\,(1 + \beta) \cos \chi \tag{32}$$

and for a three-body process

$$N_j\, K_j\, H_k = (3 - m)\,(1 + \beta) \cos \chi \tag{33}$$

For radiative recombination of O_2, no maximum of the electronic concentration can be expected if the electronic recombination coefficient depends on the concentration N_k^m with $m \geqslant 2$. The formation of an ionospheric E-layer by O_2^+ is possible, however, if the electronic recombination coefficient is proportional to the pressure ($m = 1$). The recombination of oxygen molecules may be due to the three-body process ($n = 3$) in the region where we observe the E-layer. In this case, the formation of an electronic concentration maximum is still possible by the photo ionization of O_2 even if the electronic recombination coefficient is proportional to N_k^2. Nevertheless, if we take account of the temperature effect (on α_o) in the electronic recombination coefficient (28), the effective value of m can change. For example, if α_o is directly proportional to T, the temperature, and $\beta \neq 0$ (so that T varies with height) then α varies as if α_o were constant and m were decreased to m^*, thus if m^* would be 0·91 for $\beta = 0{\cdot}1$ and 0·83 if $\beta = 0{\cdot}2$.

It should be noted here that in the case of an atmosphere with the same scale height for all constituents ($n = 1$), the formation of a layer is possible even if the electronic recombination coefficient is a function of height. For example, (31) being written

$$\frac{N_j K_j H_j}{(1+\beta)\cos\chi} = 1 - m^* \tag{34}$$

there can be a maximum of the electronic concentration if m^* is of the order 0·8 or 0·9. The above considerations lead to the result that the formation of a layer appears at a level above the maximum of the photo-ionization. Thus a scale height gradient may lead to a simple explanation of various ionospheric characteristics.

Now, the conditions imposed by the above equations for the formation of a layer determines the parameters involved in the relation

$$f_c \div (\cos\chi)^\nu$$

In an atmosphere with the same scale height for all the constituents ($n = 1$) and with an electronic recombination coefficient α which is constant, the critical frequency (22) follows the solar zenith distance as

$$f_c \div (\cos\chi)^{\frac{1+\beta}{4}} \tag{35}$$

In fact, the analysis of the exponent of cos χ cannot be represented by so simple a relation since the effect of temperature and of variable scale height can considerably modify the numerical values. For this reason, it is convenient to tabulate various possibilities for explaining the values of the exponent.

Table 1—Values of ν in the relation $f_c \div (\cos\chi)^\nu$: α_o = constant

$\alpha = \alpha_o N^m$	$n = 1$			$n = 2$			$n = 3$		
m	$\beta = 0$ limit	$= 0{\cdot}1$	$= 0{\cdot}2$	$\beta = 0$	$= 0{\cdot}1$	$= 0{\cdot}2$	$\beta = 0$	$= 0{\cdot}1$	$= 0{\cdot}2$
0	0·25	0·26	0·27	0·25	0·26	0·26	0·25	0·26	0·27
1	maximum at ∞			0·38	0·39	0·41	0·33	0·34	0·35
2	no maximum			maximum at ∞			0·42	...	...

An average of ν for Washington (1938–1948) is 0·31 and for Watheroo 0·27. If this difference is real it may be due to a scale height gradient different in two places.

If O_2^+ (by pre-ionization) is responsible for the origin of the E-region an explanation can be found even if the electronic recombination coefficient is proportional to the pressure ($m = 1$). On the other hand, if in this region the height distribution of the particles is the same as that of the main constituent so that $m = 1$, the observed values of ν can be explained only by supposing α independent of the density ($m = 0$).

The general law (31) may help, with suitable choice of the two indices m and n to explain the F_1 and F_2-regions. It is still not possible to give a definite treatment, but we can suggest the formation of a layer above the photo-ionization maximum or the formation of a layer in which there are two constituents, one ionized and one not, with different scale heights.

For the D-region in which the vertical distribution is probably the same for all the principal constituents and in which the electronic recombination coefficient is a function of height, the formula (31) indicates that a normal maximum of the electronic concentration is not possible.

Nevertheless, the absorption of short waves according to the observational results must be explained. Details are given in [3], but the view there adopted regarding the recombination coefficient is probably not correct : because it is restricted to the case $m = 0$.

Because the published experimental and observational data are meagre, we adopt various possibilities as follows:

For the recombination coefficient

$$\alpha = \alpha_0 N \, ; \; \alpha = \alpha_0 T^{\frac{1}{2}} N \text{ and } \alpha = \alpha_0 N T^{-\frac{1}{2}}$$

for scale height gradient $\beta = dH/dz$

$$\beta = 0{\cdot}1 \qquad \beta = 0{\cdot}2 \qquad \beta = 0{\cdot}3$$

and for the electronic collision frequency

$$\nu = C T^{\frac{1}{2}} N$$

The results for the variation of ω, the exponent in the absorption relation $-\log \rho \div (\cos \chi)^{\omega}$ are shown in Table 2.

Table 2—Values of ω in the relation $-\log \rho \div (\cos \chi)^{\omega}$

α	$\beta = 0{\cdot}1$	$\beta = 0{\cdot}2$	$\beta = 0{\cdot}3$
$\alpha_0 N$	0·95	0·90	0·85
$\alpha_0 N T^{\frac{1}{2}}$	0·97	0·95	0·93
$\alpha_0 N T^{-\frac{1}{2}}$	0·93	0·85	0·78

As the observed values of ω is somewhat less than 1, perhaps about 0·85, it may be concluded that β is not zero, but about 0·2.

It is practically certain that the scale height of the ionosphere increases from the D-layer upwards and it is known that the recombination coefficient varies with height; moreover, the ionizable constituents may have scale height different from that of the main constituent. It is here shown that by extending CHAPMAN's theory to allow for these circumstances, material progress can be made in the explanation of ionospheric characteristics.

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