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STRUCTURE OF THE THERMOSPHERE

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Abstract—The vertical distribution of the density in the thermosphere, deduced from satellite observations, must be explained by an increase of the scale height with altitude. A varying gradient of the scale height cannot be interpreted by assuming an increase of the temperature gradient with altitude. An examination of the interrelationships between the absolute values of density in a dark atmosphere and diurnal conditions of heat conduction reveals that the varying gradient of the scale height above 200 km is essentially due to the decrease of the molecular weight, mg, of the atmospheric constituents subject to diffusion.

In the night atmosphere the isothermy above a certain altitude (>200 km) is the critical factor characterizing the vertical distribution of density. The temperature of the isothermal region, resulting from conduction, is related to the ultra-violet heating which was available during the day. The effect of diffusion has been clearly shown by establishing a thermo-isobaric relation connecting the temperature of the isothermal region with an isobaric level where atomic oxygen has a specific concentration. From observational data on the variation of the night-time density at high levels, it is possible to deduce the variation of the temperature of the isothermal region.

The gradient of temperature in a sunlit atmosphere is related to the fraction of the ultra-violet Solar energy absorbed, which determines the magnitude of the variation of the scale height with altitude. Since heat transport is a function of the atomic or molecular concentrations and the square of the distance, it is shown that anomalies in the temperature gradient cannot be permanent.

1. INTRODUCTION

The effect of air resistance on the motion of an artificial earth satellite makes it possible to derive the atmospheric density in the region of the perigee of its orbit. Formulae relating satellite drag to orbital elements have been derived by various authors; see for example Groves[20], King-Hele[38] and Sterne[80]. Determinations of density have been made by various authors following the initial calculations of the acceleration of the first two satellites 1957 α and β (Sputniks 1 and 2), whose perigees were at about 220 km; see for example Mullard Radio Astronomy Observatory[53], Royal Aircraft Establishment[70], Sterne *et al.*[82], Sterne and Schilling[81], Harris and Jastrow[22], Jacchia[26, 29], Groves[17], Sterne[78, 79], El'Yasberg[16], Lidov[47], Mikhnevich[52], Priester *et al.*[68], Warwick[84] and Paetzold[65].

From the spring of 1958, the satellites Vanguard I (1958 β 2), Explorer I (1958 α), Explorer IV (1958 ε), and Sputnik III (1958 δ), made possible an analysis of the densities at altitudes of approximately 650 km, 350 km, 260 km and 220 km corresponding to the perigees of the above satellites. See for example, Jacchia[27], Jacchia and Briggs[34], Harris and Jastrow[23], Siry[77], Sterne[78, 79], Schilling *et al.*[74], Schilling and Whitney[72, 73], Sedov[75], King-Hele[37], Groves[19, 20, 21], Mikhnevich *et al.*[52] and Paetzold[65].

It is evident that the *absolute* values of the density which have been deduced from these observations may vary according to the methods and satellite parameters used by the various authors. Thus, the drag parameter η_S is dependent on the mass of the satellite m_S, on the effective cross-section of the satellite s_S, and on the satellite's drag coefficient C_S; $\eta_S = s_S C_S / m_S$ can differ according to the values used by the authors. Consequently, the absolute values of the density deduced can be different even if the formulae relating density to acceleration are the same. In addition, the values of the density do not necessarily relate to the same period in the life of a satellite and can correspond, therefore, to different physical states of the high atmosphere. In this case, the scale height associated with the density may not be appropriate. Finally, because the altitudes of certain satellites are not given with adequate precision the densities obtained do not correspond with the approximate altitude indicated.

From the beginning of these observations, Groves[17] stressed the importance of the equatorial bulge which changes the altitudes by 12·5 km between the equator and latitude 50°. After Jacchia[27] had indicated the existence of irregularities in the acceleration of satellite 1957 β many variations were detected and ascribed to many different causes but it was immediately obvious that the atmosphere was responsible for these irregularities. Nevertheless, the special effects were attributed to many causes such as discontinuities in the atmosphere at certain latitudes. For example, King-Hele and Walker[40] insisted that near 30°N the effect of the irregularities was caused mainly by solar disturbances. In any event, the variations of solar radiations at 20 cm and 10 cm (Priester[67], Jacchia[30]) show clearly that solar emissions play a primary role in the variation of atmospheric density. It should be noted here that it is only possible to enter into a detailed discussion of all the variations if the observational data are very precise and sufficient in number so as to be able to follow all the fluctuations as a function of altitude.

2. ANALYSIS OF OBSERVATIONAL RESULTS

2.1. *Mean values of the density*

Before carrying out an analysis of the various variations of the density it is desirable to provide a description of the results as a whole. For this reason we have summarized the main results as shown in Fig. 1.

Before being able to determine which variations modify the density we must decide on the average conditions. The different determinations shown in Fig. 1 were made for different periods, although most of them relate to the beginning of 1958. It should be noted that the rocket results, for altitudes of the order of 200 km, give variations which do not appear to agree with the results deduced from the acceleration of satellites. It appears, however, that the rocket measurements of density carried out at about 200 km lead to values of $(4 \pm 2) \times 10^{-13}$ g cm^{-3}, although certain differences can be expected since measurements made with rockets involve the collection of samples which can relate to sporadic conditions that are not necessarily representative. As a result, values of density between 200 and 250 km as given by Mikhnevich[51] are perhaps too small when compared with satellite data.

If we consider the results of LaGow *et al.*[46] and Horowitz *et al.*[25], the densities, ρ, at 200 km, at Churchill, i.e.

1956, Nov. 17, day; $\rho = 3{\cdot}6\,(^{+3}_{-1{\cdot}5}) \times 10^{-13}$ g cm^{-3}
1957, July 29, day; $\rho = (6{\cdot}7 \pm 2) \times 10^{-13}$ g cm^{-3}
1958, Oct. 31, day; $\rho = 4{\cdot}0 \times 10^{-13}$ g cm^{-3} (extrapolated)
1958, Feb. 24, night; $\rho = (1{\cdot}3 \pm 0{\cdot}6) \times 10^{-13}$ g cm^{-3}

it is clear that the median value corresponds to the value obtained from the satellite data, $(4 \pm 2) \times 10^{-13}$ g cm^{-3}. Furthermore, the value obtained at White Sands, of the order of $1{\cdot}4 \times 10^{-13}$ g cm^{-3} at 200 km, corresponds to an atmosphere, in 1951, for which the pressure was only 10^{-4} mm Hg at 100 km when it was $(3 \pm 1) \times 10^{-4}$ mm Hg at Churchill.

Mikhnevich[51] indicating a density of $2{\cdot}7 \times 10^{-13}$ g cm^{-3} at 200 km is still inside of the possible variation. However, he adopts[52] the value of $2{\cdot}1 \times 10^{-13}$ g cm^{-3} at 225 km for an atmospheric model.

It can be concluded that an average value of the order of 4×10^{-13} g cm^{-3} represents the atmospheric density at 200 km during the sunspot maximum in 1958–1959. Variations leading to $(4 \pm 2) \times 10^{-13}$ g cm^{-3} can be accepted, Extreme values between 1 and 7×10^{-13} g cm^{-3} are not representative of latitudinal, seasonal or diurnal variations at 200 km, but should be associated with the effect of solar activity if they are not included in the possible errors of measurement.

It must be pointed out that an analysis of the varying conditions at 200 km is to be related to the analysis of conditions between 100 km and 150 km. For example, there is a variation of the density of molecular oxygen by a factor of 3 according to the measurements made by Byram *et al.*[6] and Kupperian *et al.*[45]. Like-

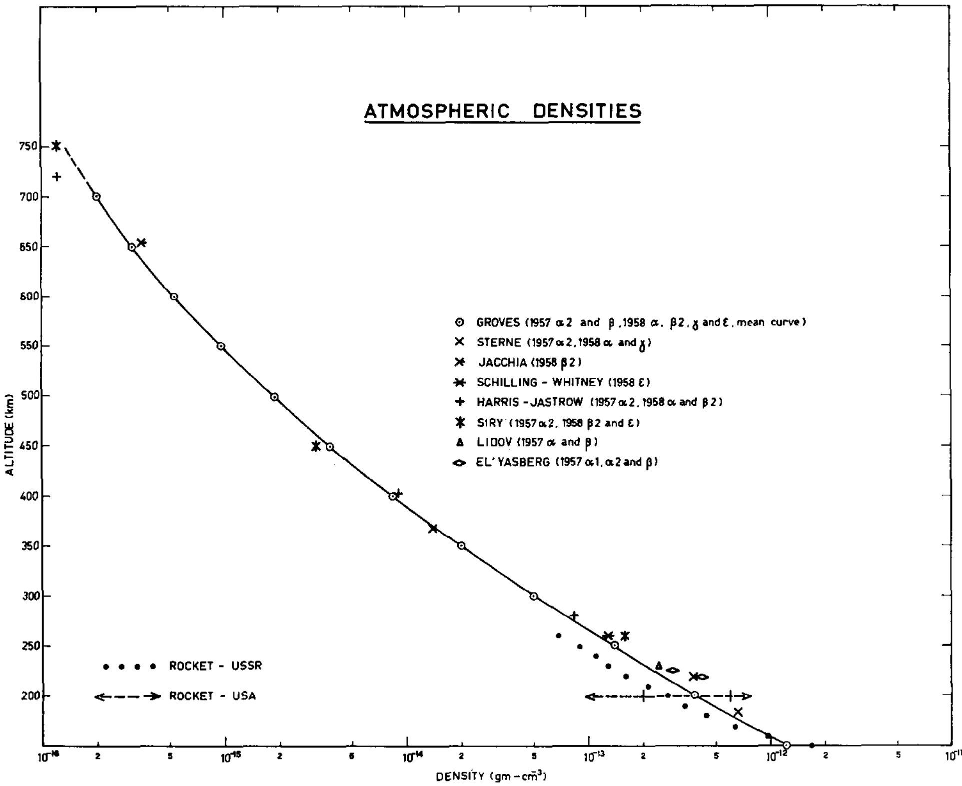

Fig. 1. Density–altitude relation above 150 km from various determinations. The average curve represented by Groves's data shows that the height variation of the density can only be explained by an increase of the scale height with altitude. Measurements made in 1957 and first months of 1958.

wise considering the results obtained by Horowitz and LaGow[24] at White Sands and by Horowitz *et al.*[25] at Ft. Churchill it is clear that there is a broad range of a factor 4 in the pressure and density data at 100 km. In other words if the value $(2{\cdot}5 \pm 1{\cdot}5) \times 10^{-4}$ mm Hg is accepted for the pressure at 100 km, the possibility exists that the density at 200 km is subject to a variation of 50 per cent even if the structure of the atmosphere above 100 km is essentially the same. Therefore, the data available on atmospheric density obtained by means of rockets and satellites show that variations of the density near 200 km must be connected with the atmospheric structure in the entire region between 100 and 200 km. A certain variation of the density at 200 km must be explained by the boundary conditions near 100 km and by the atmospheric structure above the later altitude.

The curve in Fig. 1 has been drawn following the determination of Groves[19] and can be considered as an average distribution for the first six months of 1958 corresponding to a certain sunlit atmosphere. The absolute values near 200 km depend on the value which is assumed for the scale height at that altitude. For example, the results of Lidov[47] lead to values for ρ between $2{\cdot}4 \times 10^{-13}$ g cm^{-3} to $3{\cdot}2 \times 10^{-13}$ g cm^{-3} if the scale heights are 50 km and 30 km, respectively. El'Yasberg[16] has adopted $H = 25$ km at 225 km. The average value of Groves[19] at 200 km is 46 km. Such differences show that variations do occur in the atmosphere above 200 km. But the main conclusion to be drawn from the average values of the density is that the vertical distribution must be explained by an increase of the scale height H with altitude. In fact the equation for a perfect gas and the static equation indicate[57] that the density can be expressed by the relation

$$\frac{d\rho g}{\rho g} = -\frac{1+\beta}{\beta}\frac{dH}{H} \qquad (2.1)$$

where $\beta = dH/dz$ is the gradient of the scale height and g is the gravitational acceleration. The integration of equation (2.1) for a height interval sufficiently small so that β can be considered practically constant leads to

$$\frac{\rho g}{\rho_0 g_0} = \exp\left[-\frac{(1+\beta)z}{\frac{1}{2}(H+H_0)}\left\{1+\frac{1}{3}\left(\frac{\beta z}{H_0+H}\right)^2+\dots\right\}\right] \qquad (2.2)$$

In using equation (2.2) we can see that H increases regularly with altitude between 150 km and 700 km. Thus the results of Groves[19], corresponding to a certain sunlit atmosphere in 1958, coincide with an average model presented by Nicolet[58]. Subsequent calculations carried out by Nicolet[59, 60] involving an analysis of the physical conditions of an atmosphere in which the temperature is constant above 220 km lead to the important conclusion that: *the vertical distribution of the density of the high atmosphere can be explained if the atmosphere in which the various constituents are subject to diffusion is isothermal above a certain level.* Such an atmospheric model shows that ultraviolet radiation is involved in the heating of the atmosphere.

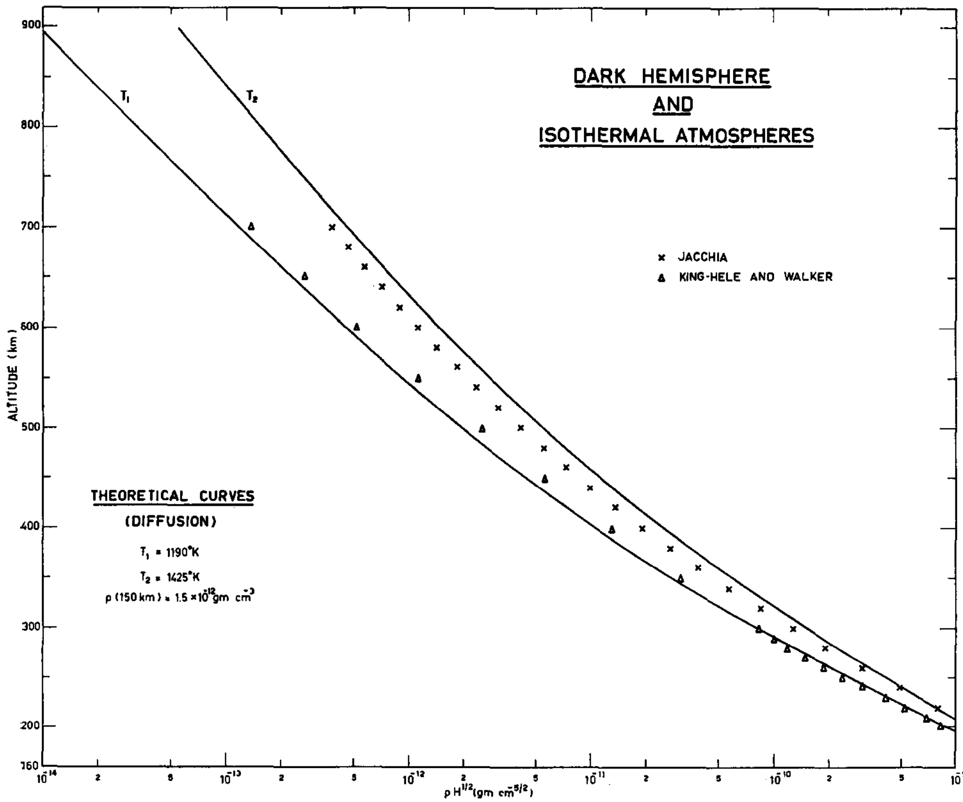

Fig. 2. $\rho H^{1/2}$ proportional to orbital acceleration of satellites in an isothermal atmosphere subject to diffusion. The density–altitude relations deduced from observational data can be followed. It is possible to fit the satellite observations of density, corresponding to a night-time atmosphere by an isothermal atmosphere.

A recent analysis made by King-Hele and Walker[42] leads to a night-time distribution in 1959 shown in Fig. 2. This figure further shows that an isothermal atmosphere at about 1200°K with diffusion beginning at 150 km can represent the observations. The observational results do not differ from the model computed by Nicolet[60]. In the same way, an analysis of night-time conditions by Jacchia[33] (see Fig. 2) also shows the possibility of following the atmospheric distribution for high solar activity when a temperature of the order of 1400°K is adopted for the isothermal layer.

The above results lead to the conclusion that there is a thermopause and that its level is subject to a diurnal variation. Its altitude is maximum in a sunlit atmosphere and is minimum in a dark atmosphere. However, when the scale heights corresponding to isothermal atmospheres in diffusion equilibrium are compared (Fig. 3) with the empirical scale heights deduced by King-Hele and Walker[42] and by Jacchia[33] considerable differences are found. King-Hele and Walker have introduced a negative gradient below 250 km and Jacchia's curve corresponds to an important gradient even at 600 km. Such differences show that arbitrary atmospheric models can be made to follow

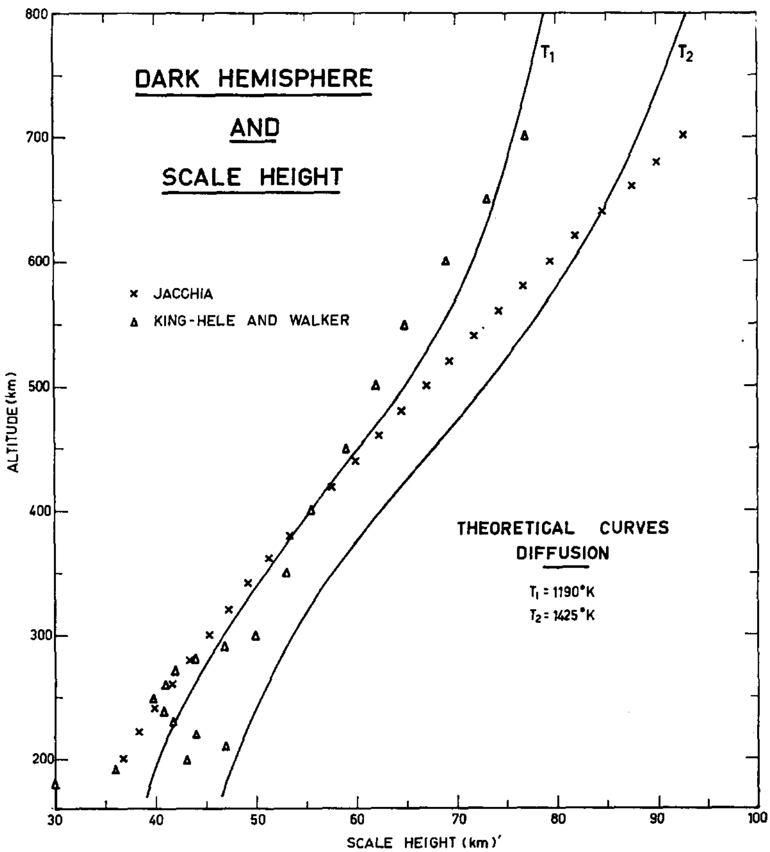

Fig. 3. Variations of the scale height with altitude. The variation of the scale height is associated with a variation of the mean molecular mass depending on diffusion. In this figure the scale height–altitude relations must be associated with the density distributions shown in Fig. 2. It is possible to deduce from observational data of $\rho H^{1/2}$ inconsistent values of the scale height.

atmospheric densities deduced from satellite observations.

2.2. *Variations of the density*

Jacchia[32] has shown that the amplitude of the fluctuations of the accelerations, proportional to $\rho H^{1/2}$, increases with the altitude of the perigee of the satellite. In fact, the important fluctuations appear simultaneously at all altitudes and the diurnal variations are magnified between 350 and 650 km.

The effect of the earth's equatorial bulge is evident. Each transit at the equator of Explorer I corresponds to a maximum and the change in perigee height of Sputnik III leads to a variation in the acceleration.

In Fig. 4 we consider data from satellite

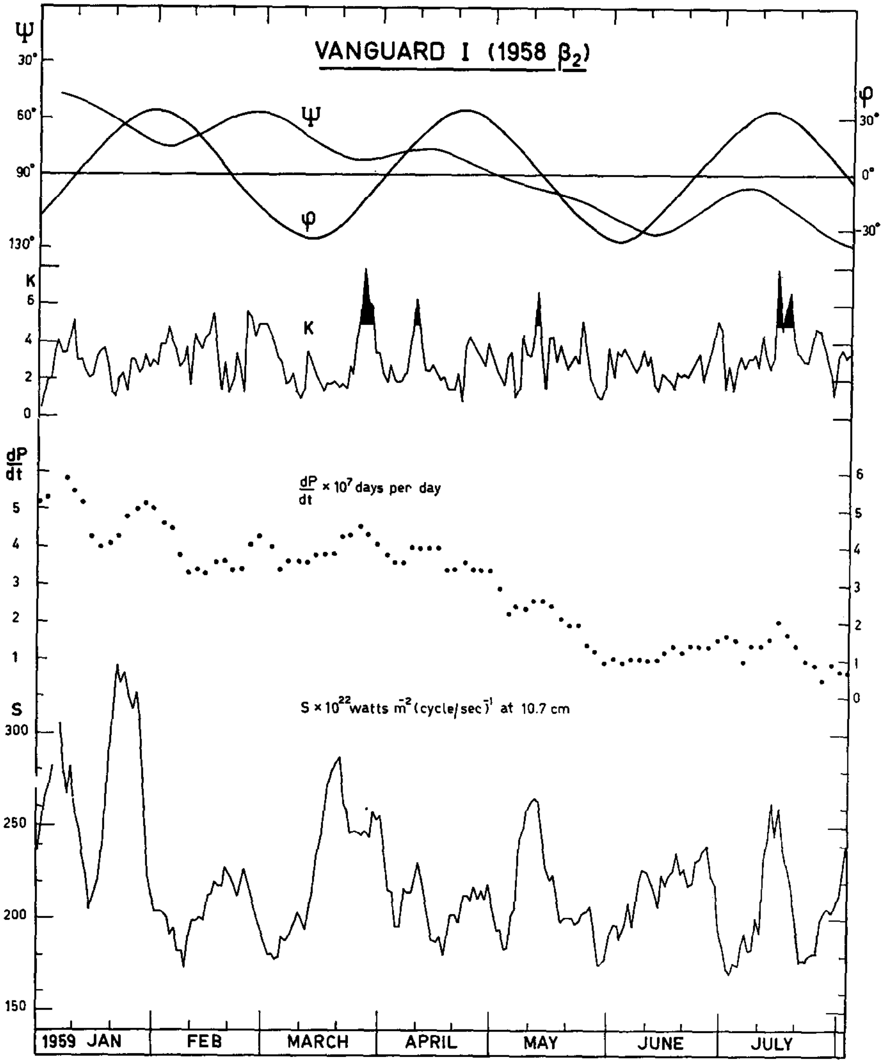

Fig. 4. Orbital acceleration of Vanguard I (1958 β2). $-dP/dt \propto \rho H^{1/2}$ [5]. Perigee latitudes (φ) and angular distance of the perigee from the subsolar point (Ψ)[32]. The solar activity is represented by daily values of solar radiation at 10·7 cm (Ottawa, Covington) and the magnetic activity by the daily K values deduced from the three-hourly indices[2].

1958 β 2 during the first half of 1959, obtained from the values determined by Jacchia[32] and Briggs[5]. The solar activity is represented by daily values of solar radiation at 10·7 cm as observed in Canada (National Research Council) and by the daily values deduced from the three-hourly K indices provided by Bartels. It is obvious, as Jacchia has shown[32], that there is a very close correlation between the variations of the density and those of the solar radiation as obtained using the electromagnetic radiation at 10·7 cm. As for a magnetic activity relation to the corpuscular radiation, there is no general correspondence although, in certain cases, it is possible to see that corpuscular effects can occur during important geomagnetic disturbances. It appears, however, that a greater resolution in the orbital acceleration is needed to detect short-lived perturbations.

On April 30 the density has decreased (Fig. 4) when the sun is at 90° (the angle at the centre of the earth determined by the position of the sun and of the perigee of the satellite); that is, when the earth below the perigee is no longer sunlit. Finally, on July 31 the nocturnal effect is complete and the density is very low. These data show that the atmosphere at an altitude of 650 km is subject to a very important diurnal variation and the different analyses of Jacchia[32], Wyatt[86] and of Priester and Martin[69] show that the diurnal effect is the principal one.

This strong diurnal variation is confirmed by the observed accelerations of Vanguard II 1959 α[33], for which the diurnal variation of $\rho H^{1/2}$ is of the order of a factor of 5 between March and August 1959 (Fig. 5). It is therefore necessary to consider that the altitude of the thermopause and *the temperature of the isothermal atmosphere vary considerably between night and day.* Under such conditions the *normal heating of the upper atmosphere* takes place by electromagnetic radiation. It is, therefore, not necessary to look for a corpuscular effect[2] when investigating such a normal heating effect of the atmosphere, a hydromagnetic effect[14, 15] or any effect other than that of electromagnetic radiation, because such effects cannot be associated with a diurnal variation whose character is so pronounced at 650 km. Such effects must be linked with disturbances.

The variations of $\rho H^{1/2}$ at an altitude of about 350 km, corresponding to the perigee of Explorer I (1958 α), are considerably smaller than at an altitude of 650 km. From the beginning of January 1959, when the angular distance of the sun reaches 180° until it reaches 90° at the March equinox, the density decreases slightly in agreement with the diminution in solar flux. Between March and August 1959 the increase in the acceleration occurs with the fluctuations clearly associated with the sequence of perigees at the equator even if some association can be made with the solar activity.

Thus, at an altitude of 350 km, the effect of solar heating is evident and leads to a clear diurnal variation. The effect of the earth's equatorial bulge is of the order of 10 per cent.

When an analysis of the variations of the density is carried out at altitudes less than 300 km it is found that the diurnal effects are greatly diminished. In the analysis of the density deduced from the acceleration of 1958 ε[73] the diurnal effects that can be inferred for the period between July and November 1958 reach a maximum of 20 per cent; at the same time, an apparent latitude effect, caused by a change in perigee height due to the earth's equatorial bulge, is certainly of the order of 20 per cent. The discontinuities in the density at 50°N and S cannot be attributed to a latitudinal variation in air density.

The analysis of the fluctuations of the Sputniks by various authors already mentioned, in particular King-Hele[38], have shown that the diurnal effect is generally masked by other effects. In Fig. 6 the data given by Kozai[43] for 1958 δ 2 are shown. It may be seen that from January to November 1959 the maximum variation is ± 60 per cent. The main character of this variation corresponds to a displacement of the perigee from 24°S on 1 January 1959 to 65°S, at the beginning of June, and to less than 10°S at the end of November 1959. Thus, the perigee was located in the Antarctic during the winter. Because of this, the altitude of the perigee did not diminish with time[49]. In June its altitude, about 225 km, was a maximum and it is necessary to consider an increase in altitude

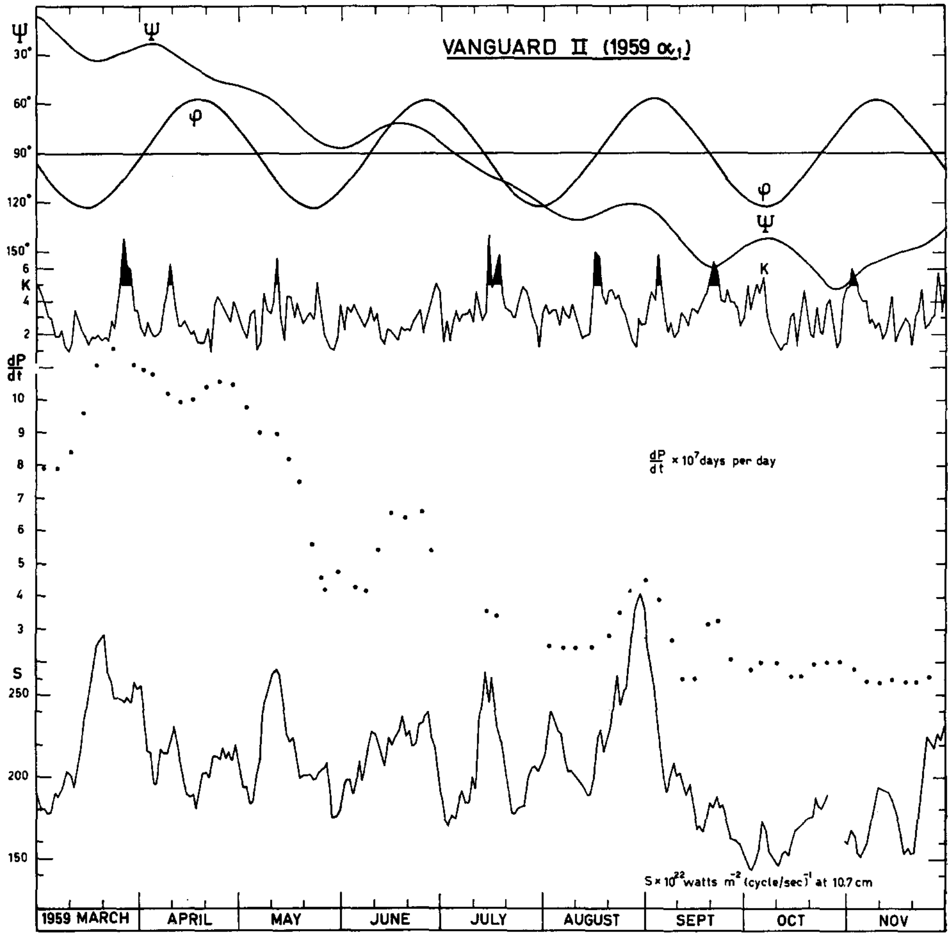

Fig. 5. Orbital acceleration of Vanguard II (1959 α). $-dP/dt \propto \rho H^{1/2}$ [33]. Other symbols, see Fig. 4.

of some 18 km with regards to the equator. As a consequence a decrease (30 per cent to 40 per cent) of dP/dt (see Fig. 6) can be explained by the flattening of the earth. It is, therefore, clear that the general variation observed in the acceleration of this satellite is due to change in the altitude of the perigee and not to a discontinuity caused by a latitude effect.

Another remarkable result is the very close association with geomagnetic storms that Jacchia[31] discovered for satellite 1958 δ 1, thanks to a resolution of 10 revolutions in the analysis of the acceleration. Fig. 7 shows the variation of the acceleration of 1958 δ 1 as given by Jacchia[31] with two remarkable peaks on 9–10 July and 4–5 September when a resolution of 10 revolutions is used in the analysis of observations. It is clear that such an increase of the atmospheric density is associated with the magnetic activity defined by the *K* indices. A remarkable fact that Jacchia[31] has found is that the latitudes of the perigee of 1958 δ 1 were 35°N and 15°N in 9 July and 4 September, respectively. Furthermore, it is of interest to note that the angular distances, Ψ, of the perigee from the sub-solar point were 120° and 75°, respectively. Finally, it is clear that the electromagnetic solar radiation represented by 10·7 cm was decreasing after the occurrence of its peak two or three days before. These two events

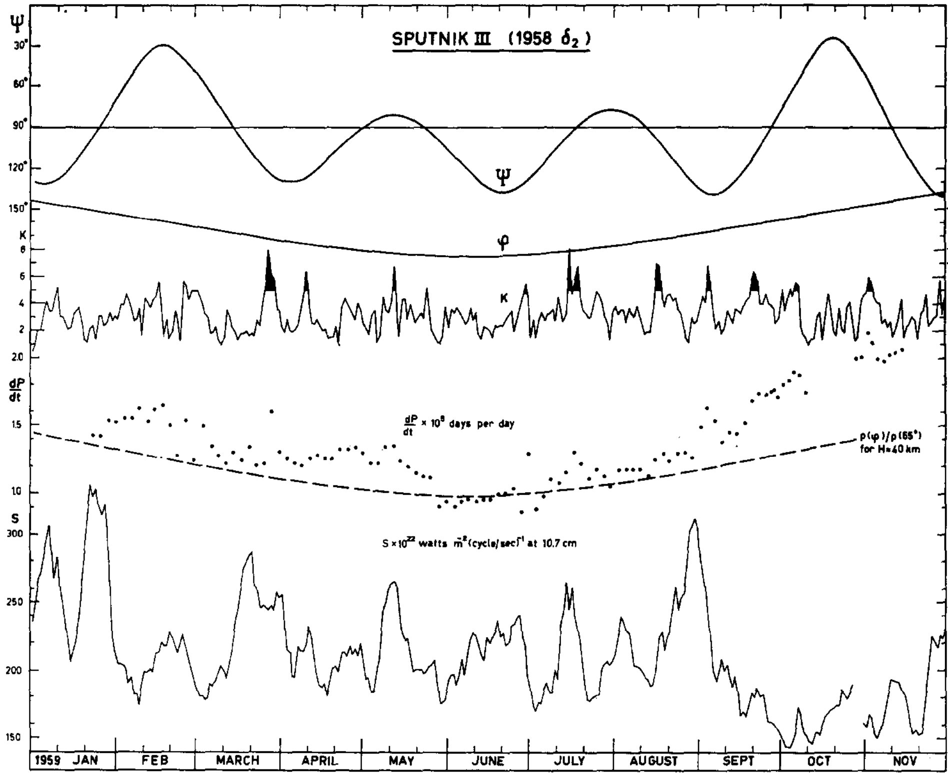

Fig. 6. Orbital acceleration of Sputnik III (1958 δ2). $-dP/dt \propto \rho H^{1/2}$ [43]. Other symbols, see Fig. 4.

show that a limited resolution corresponding to several days may smooth the transient disturbances affecting the atmospheric density.

However, if we consider the periods when the perigee was in a dark atmosphere (Ψ in Fig. 7) we can see three other increases associated with the K indices ($K > 5$) even if the resolution is only 25 revolutions. They are 28–30 June, 24–26 September and 22–24 October, when the angular distance, Ψ, of the perigee from the subsolar point is greater than 90°. Again the corresponding latitudes of the perigee are very low; between 15°N and the equator from September to October.

Taking again the Smithsonian data for Sputnik II (1957 β)[29], it can be shown (Fig. 8) that certain variations of the density can be associated with $K \geqslant 5$ indices. Two remarkable associations are found on 1–2 January and 9–11 February when the Ψ angles were 80° and 120° and the perigee latitudes were 30°N and 15°N, respectively.

Even if the observations of 1958 δ 2 are irregularly distributed three pronounced increases in the acceleration curve of Sputnik III (1958 δ 2) can be seen (Fig. 6) for 27 March, 29 June and 4 September. These are associated with geomagnetic disturbances which are represented by the K indices[2]. It should be noted that these three remarkable increases in the atmospheric density were observed during the three periods when the perigee was in the dark atmosphere. This shows again that the reactions of the atmosphere to a "corpuscular effect" are more clearly distinguishable when effects on a sunlit atmosphere can be eliminated.

If we consider periods during which the atmosphere was sunlit (May, July, August) important magnetic disturbances were also observed, but their associated effects do not

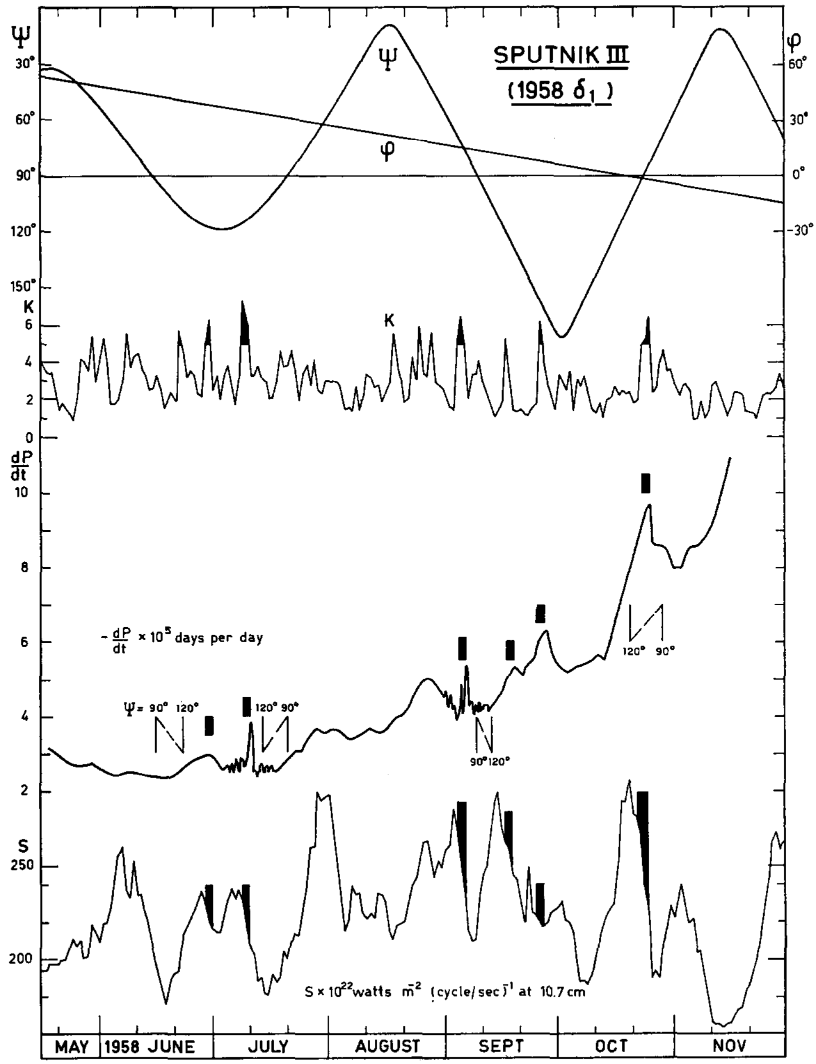

Fig. 7. Orbital acceleration of Sputnik III (1958 δ 1). $-dP/dt \propto \rho H^{1/2}$ [31]. Other symbols, see Fig. 4.

seem to be so disturbing to the behaviour of the acceleration of satellite 1958 δ 2. In particular, the magnetic storm of 12 May 1959 (see the *K* indices Fig. 6) observed by Ney *et al.*[54] in association with cosmic radiation does not show, with the low resolution of the acceleration curves, a special effect on the acceleration of the satellite. The maximum in the acceleration curve, about 8 May, is more closely associated with the maximum of the solar electromagnetic radiation. Similarly, the maximum near 15 July at sunrise appears to be related to the electromagnetic solar radiation, although the cosmic-ray bursts described by Winckler[85] are noteworthy.

Moreover, a comparison should be made between the magnetic storm of 16 August 1959 and the recurrent storm of 4 September (during

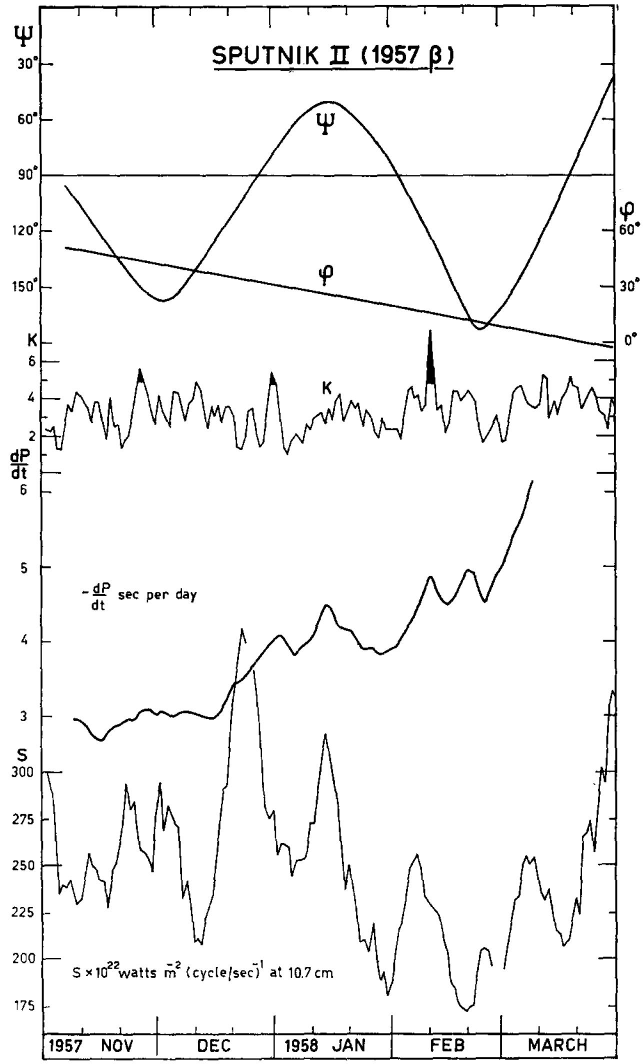

Fig. 8. Orbital acceleration of Sputnik II (1957 β). $-dP/dt \propto \rho H^{1/2}$ (29). Other symbols, see Fig. 4.

the night for 1958 δ 2). During the first 24 hr of the magnetic storm of 16 August, Arnoldy *et al.*[1] found that about 3/4 of the particles in the outer Van Allen belt had been removed and had penetrated into the atmosphere. But the effect of the "corpuscular" heating of the atmosphere is probably not greater than the electromagnetic heating, whilst the exceptional effect observed on 4 September is clearly a transient heating in the dark atmosphere.

These different examples show that it should be possible to distinguish between all of the external effects by a detailed determination, with sufficient resolution, of the acceleration of the satellites. In any event, the diurnal variation and the 28-day periodicity show that the absorption of solar electromagnetic radiation is the primary process for heating the atmosphere above 100 km. The "corpuscular" radiation with its associated processes can easily affect the nocturnal conditions but is more difficult to detect in a sunlit atmosphere which is already heated by an increased electromagnetic radiation. Furthermore, the strong diurnal variations at the higher altitudes can mask such effects. Finally, the penetration of energy below 100 km has practically no influence on the structure of the thermosphere in that the energy involved in flares is not important compared with the total kinetic energy of the atmosphere. The most remarkable phenomenon near 200 km is the relation with the energy of the solar radiation which varies in accordance with changes in the solar activity. This is the reason that several investigators have found correlations with various indices of solar activity. Such a variation at 200 km means that the atmosphere between 100 and 200 km is affected and must correspond to a general increase of the scale height in the entire layer.

The variations of the density as a function of altitude demonstrate that the diurnal variation is magnified with increasing height. It is the dominant factor at higher altitudes and must be associated with the gradient of the scale height and altitude of the thermopause.

The variation from one day to another is closely associated with solar activity and the size of the variations of the density, increasing with the altitude, depends on the energy of the solar electromagnetic radiation which is available during a 27-day period.

A "corpuscular effect"* is evident at the time of magnetic disturbances but it is only introduced sporadically and the energies which are involved are generally less than ultra-violet energies. Seasonal and latitude effects can only be secondary in relation to the complex effects of the diurnal and solar variations.

3. THE CONSTITUTION OF THE THERMOSPHERE

The constitution of the thermosphere, that is to say of the atmosphere above 85 km, theoretically depends on the state of molecular dissociation at the highest altitudes. Indeed, the essential observation that the scale height H increases with altitude necessitates an analysis of the variations of three parameters: the temperature T, the mean molecular mass m and the acceleration due to gravity, g.

If the atmosphere above a certain altitude was in a state of complete dissociation, an increase of H with altitude could only be explained by an increase of temperature with altitude. Above 300 km, we can neglect an important heating effect by ultra-violet radiation in that there is practically no absorption of such radiations at very high altitudes. In this case, a very pronounced dissociation of molecular oxygen below 200 km, with a low pressure (10^{-4} mm Hg) at 100 km, leads to an atomic oxygen atmosphere. To explain the gradient of the scale height it is necessary to introduce a flux of external heat transported by conduction. Such an application has been made by Nicolet[55, 57] using Chapman's theory of an extension of the solar corona.

But an atmospheric model in which the pressure at 100 km reaches 3×10^{-4} mm Hg, with a small percentage of the oxygen dissociated, represents more closely the atmospheric conditions leading to a further understanding of the thermosphere. An important temperature gradient exists below 200 km and leads to a large abundance of molecular nitrogen at high altitudes[58, 59]. Since atomic nitrogen is a secondary constituent[56, 57] we may consider an atmosphere whose constitution depends essentially on the ratio N_2/O.

In a general study, Nicolet[59] has shown how the problem of the conditions in the thermosphere can be analysed. Using as an example the following conditions at 100 km:

$p = 3 \times 10^{-4}$ mm Hg; $T = 200°$K;

$$\rho = 6{\cdot}6 \times 10^{-10}\ \mathrm{g\ cm^{-3}} \tag{3.1}$$

corresponding to the concentrations (cm^{-3})

$$n(O_2) = 2{\cdot}2 \times 10^{12},\ n(N_2) = 1{\cdot}1 \times 10^{13};$$
$$n(O) = 1{\cdot}4 \times 10^{12} \text{ and } M = 27{\cdot}4 \tag{3.2}$$

the conditions at higher altitudes can be determined if we fix the level for the beginning of diffusion and the gradient of the scale height.

In Fig. 9 are shown conditions such that diffusion begins at 120 km and at 150 km, and the scale height has large gradients of $\beta = 1{\cdot}0$,

* "Corpuscular effect" means no direct electromagnetic radiation effect.

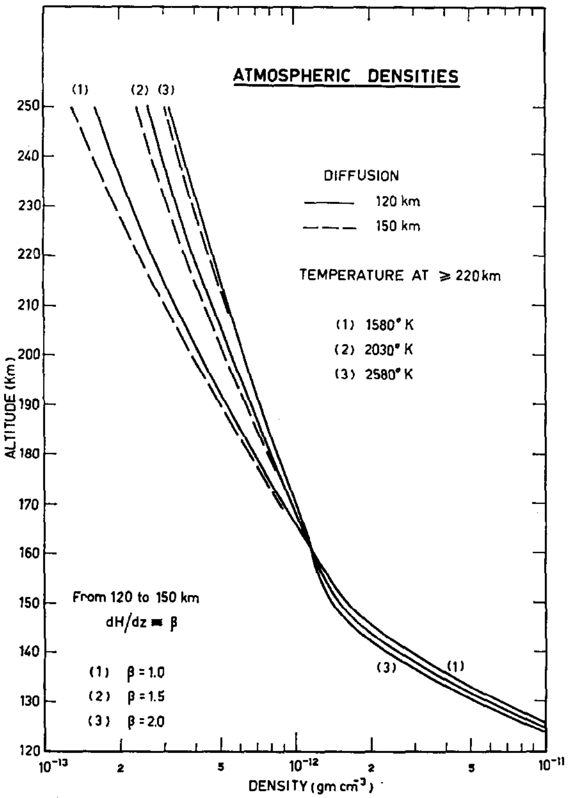

Fig. 9. Density–altitude relations above 120 km for various gradients of temperature. The height variation of the density shows that any solution for an inadequately short range of altitudes is arbitrary even if the beginning of the diffusion is introduced at 120 km or 150 km. Boundary conditions being fixed at 120 km (T and ρ constant), variation of density at 200 km is small.

1·5 or even 2·0 between 120 and 150 km and an arbitrary gradient of $\beta = 0{\cdot}2$ between 150 and 220 km. In order to simplify the calculations it was noted that these two gradients are almost equivalent to a variable gradient diminishing with height until about 300 km.

Considering Fig. 9 we see that very different conditions do not modify to any great extent the vertical distribution of the density. For example, the density at 200 km only varies by ± 25 per cent from the mean value for the above range of variables. Consequently we can conclude that, for constant conditions at 100 km, it is easily possible to obtain the densities, deduced from the satellite observations if we consider that there is a large temperature gradient between 100 and 200 km. Furthermore, the vertical distribution of the density and the absolute value at 200 km are practically independent of the exact value of the temperature gradient. An increase of the order of 50 per cent in the density at 200 km would require an increase of the order of 1000°K in the temperature. An increase of the density at 200 km should be caused mainly by its increase in the region of 120 km.

A similar effect explains the differences in the observations made with rockets at different periods at White Sands and Ft. Churchill. Similarly, differences obtained at the same place of observation can only result from a change of temperature in the lower thermosphere. In other words, the structure of the thermosphere depends primarily on the conditions at the limits applicable to the lower thermosphere (different densities at 100 km). Moreover the fact of having introduced diffusion at 120 km or 150 km does not modify the above conclusions for the atmosphere at altitudes of less than 250 km.

We can therefore conclude that: *the density of the high atmosphere is essentially dependent on the solar energy absorbed below 200 km.* This energy determines the temperature gradient up to 400 km in a sunlit atmosphere and fixes the temperature of the isothermal atmosphere up to the highest altitudes.

Since various temperature gradients between 120 km (E-layer) and 200 km (F-layer) modify only slightly the values of the density at 200 km, if constant boundary conditions are taken at 120 km, the small variations in the densities deduced from satellite observations near 200 km are easily explained. But it must be pointed out that variations must occur in the E-layer which modify the boundary conditions for the whole thermosphere. The computations lead to the following results:

$$120\text{ km},\ \rho = 3{\cdot}26 \times 10^{-11}\text{ g cm}^{-3};\quad T = 262\text{°K} \tag{3.3}$$

$$150\text{ km},\ \rho = (1{\cdot}5 \pm 0{\cdot}03) \times 10^{-12}\text{ g cm}^{-3};\quad 725\text{°K} \leqslant T \leqslant 1650\text{°K} \tag{3.4}$$

Thus, it is certain that varying conditions occur inside the E-layer.

The total kinetic energy varies between 5×10^4 and 5×10^5 ergs cm^{-2} from about 120 km to 100 km. If an energy of the order of 1 erg cm^{-2} sec^{-1} is available for the heating of the E-layer, it is clear that the boundary conditions at 120 km will be subject to variations resulting from solar activity. In the same way, the region of the $F1$-layer where the total kinetic energy is less than 5×10^4 erg cm^{-2}, will be strongly affected by an ultra-violet heating of the order of 1 erg cm^{-2} sec^{-1}.

Below 100 km, at 85 km for example, the kinetic energy of a vertical column is of the order of 5×10^6 erg cm^{-2} and an equivalent solar energy during one day of 12 hr would require a total absorption of about 100 erg cm^{-2} sec^{-1}. We can therefore conclude that the entire thermosphere above 100 km is essentially dependent on the solar energy and its fluctuations if the energy available for heating is of the order of 1 erg cm^{-2} sec^{-1}. Such an energy input must be found in the X-ray and ultra-violet radiations absorbed above 100 km.

4. SOLAR RADIATION

The diurnal variation of the atmospheric density above 250 km shows clearly that the heating of the thermosphere depends mainly on the electromagnetic radiation absorbed in the atmosphere above 100 km.

Above 100 km the atmosphere absorbs radiations with wavelengths of less than 1750 Å as a result of the dissociation of the oxygen molecules. The recombination of oxygen atoms mainly occurs below 100 km after a downward transport and for this reason there is no important heating available from *in situ* recombinations. However, the energy available from the various monochromatic radiations between 1500 Å and 1300Å corresponds to about 3 ergs cm^{-2} sec^{-1}, and the part transformed immediately into heat is about 0·5 erg cm^{-2} sec^{-1} in the E-layer. Furthermore, the total energy from X-rays absorbed in the E-layer does not exceed 0·5 erg cm^{-2} sec^{-1} at the maximum of the solar cycle. In addition, the heat flow from upper levels will lead to an important effect in the E-layer since convection and conduction are certainly involved. We can conclude therefore that the E-layer is subject to varying conditions depending on the coronal flux in the X-ray spectrum and on the chromospheric and coronal flux in the ultra-violet spectrum.

At altitudes above the E-layer, from the beginning of the $F1$-layer, we must take into account the absorption of radiations of wavelengths longer than 200 Å. This corresponds to the entire ultra-violet spectrum involving helium lines at 304 Å and 584 Å and also coronal lines of highly ionized atoms. An energy of the order of 1 erg cm^{-2} sec^{-1} can produce temperature gradients of the order of 20°K per km. Such important gradients lead to very high temperatures above 200 km.

If we consider a steady state for an overhead sun, it is easy to show (see Section 5) that the energy E is distributed with height as follows

$$E = E_{\infty} + E_{uv}(1 - e^{-\tau}) \qquad (4.1)$$

in which E_{∞} denotes the energy flow at the top of the layer, E_{uv} the ultra-violet solar energy available at the top of the earth's atmosphere for a certain spectral range and τ is the optical depth, which is defined by

$$\tau = K_{\lambda} \int_{z}^{\infty} n dz \qquad (4.2)$$

Since the heating must be proportional to the energy absorbed, it is evident that the temperature gradient will decrease with height up to a certain altitude where the optical depth is negligible.

If there is no external heat flow, the sunlit atmosphere must be isothermal where there is no absorption of solar radiation. Since X-rays and ultra-violet radiations are absorbed between the E-layer and the F-layer the temperature must increase continuously and no permanent decrease can be expected in the whole thermosphere.

In order to study the variation of the short ultra-violet radiations and of X-rays one refers to measurements of the centimeter or decimeter electromagnetic radiations emitted by the solar corona. Nicolet[60] has compared the results obtained at various observatories from 3 to 30 cm and has shown that the general behavior of the solar activity is the same for the entire spectral region. The mean daily solar fluxes from 8 to 15 cm are subject to identical variations. The extremes of 3 cm and 30 cm differ from 10 cm by an extent which is much less than the fluctuations. In other words, the largest fluctuations of the solar radiation from 3 cm to 30 cm are observed at about 10 cm although the complete spectrum varies, under the same conditions, with the solar activity.

Since Priester and Martin(69), and Jacchia(33) after having used 10 cm(32), have based their analysis of satellite accelerations on the solar energy of 20 cm wavelength observed at Berlin, it is important to compare the observational data at 10 cm and 20 cm. Fig. 10 shows that, in 1958, the solar radiation at 10·7 cm measured at Ottawa does not indicate a general variation which is as pronounced as that for the solar radiation at 20 cm measured at Berlin. In fact, during 1958, the relation between the extremes of radiation at 20 cm is of the order of 5 while that for 10·7 cm (which must be a maximum) is only about 2. Since Priester and Martin(69) and Jacchia(33) are able to follow the general variation of the atmospheric densities deduced from satellite measurements by using the 1958 measurements at 20 cm, it is quite strange that such a good correlation has been obtained. There is no way to account for a special behavior at 20 cm, with a variation of a factor of 5 in 1958, when the entire spectrum between 3 cm and 30 cm does not vary by more than a factor of 2.

Fig. 11 shows that, in 1959, the solar radiations measured at 10·7 cm in Ottawa and at 20 cm in Berlin are subject to the same general variations and that the ratio Berlin/Ottawa of the solar fluxes, taking as a unit the mean value for 1959, does not vary by more than ±20 per

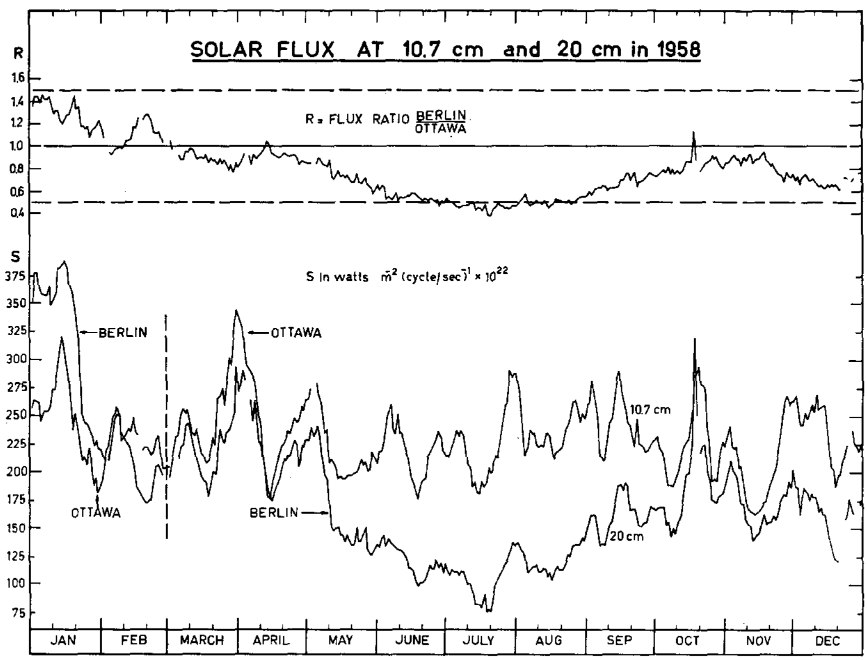

Fig. 10. Comparison between solar fluxes at 10·7 cm (Ottawa) and at 20 cm (Berlin) in 1958. While the difference between maximum and minimum corresponds to a factor of the order of 5 at Berlin, it is only of the order of a factor 2 at Ottawa. The real variation cannot be more than a factor of 2 and the flux ratio of 20 cm/10 cm cannot vary from 1·5 to 0·4.

cent. The data on radio radiations at 20 cm, therefore, introduce a systematic error when used in 1958 to compare the variations of the atmospheric density with that of the solar flux.

In conclusion, when errors in calibration of radio measurements have been eliminated one can say that in general the variations of the activity are represented by all wavelengths between 3 and 30 cm. The most pronounced range of the variations is given by the spectral region centered on 10 cm. Thus the observations at Ottawa at 10·7 cm, made during 1958 and 1959, and indicating a maximum variation of a factor 2·2, fix the maximum of the variation of the mean daily value for all the spectral region from 3 cm to 30 cm, i.e. of the slowly varying component of the sun's radio-emission.

Since the temperature and its vertical distribution in the chromosphere and corona remain normal in the active regions emitting the decimeter radiations, it is convenient to consider as the basic radiation of the sun heating the upper atmosphere the minimum value of the radio flux. Hence we may assume that from July to November 1958 the basic radiation S at 10·7 cm was never less than 220 units [watts m^{-2} (cycle-sec)$^{-1} \times 10^{-22}$], while in 1959, from the beginning of September to the end of November, it was less than 220 reaching 150 at the beginning of September.

Such differences must correspond to variations of the solar emission in the ultra-violet region and particularly in the X-ray spectrum. It is evident that the variation in amplitude of the radio emission cannot be directly proportional to that of the entire ultra-violet spectrum or of the spectrum at the shortest wavelengths. The difficulty of obtaining an exact relation between the optical and radio ranges is easy to understand since such radiations originate between the photosphere and corona, i.e. from the lower and cooler part of the solar atmos-

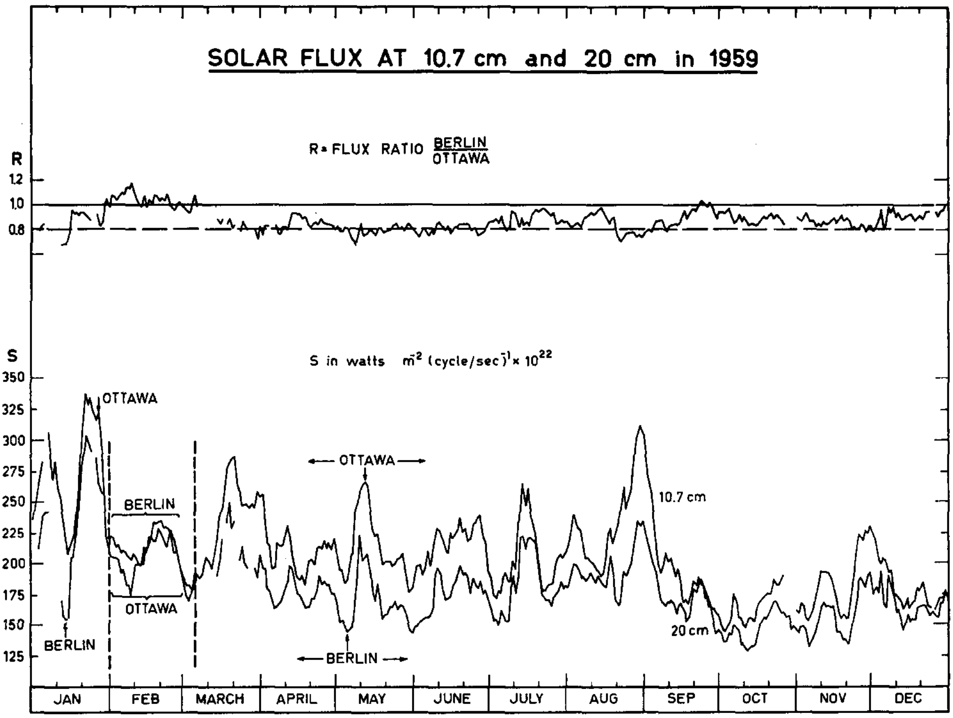

Fig. 11. Comparison between solar fluxes at 10·7 cm (Ottawa) and at 20 cm (Berlin) in 1959. The flux ratio 20 cm/10 cm varies only about ±20 per cent and such a small variation indicates that 1958 results on 20 cm cannot be accepted.

phere up to the normal high temperature corona. Nevertheless, it is true that the terrestrial atmosphere between 100 km and 200 km is subject to the variable heating resulting from the absorption of ultra-violet radiation and X-rays. The implication is that the thermosphere is heated by all radiations for which the absorption cross-section is greater than 10^{-19} cm^2. There is no particular terrestrial layer above 100 km which is especially heated, since the energy is distributed with height according to (4.1).

Sources of heating other than electromagnetic radiation depend on the energy which is available. The use of the energy of the radiation belt in one form or another is limited by the total energy available. This, according to Dessler and Vestine[15], is 6×10^{22} erg and is about a factor of 10 greater than the energy involved in a magnetic storm; 10^{22} ergs according to Chapman and Bartels[10]. On the other hand, Arnoldy *et al.*[1] have observed, during the first phase of a geomagnetic storm, a loss of about two-thirds of the energy of the outer belt. Such energies place limits on the energy which would be immediately available from the radiation belt and we agree with Bates[4] that the time period would be too short if "corpuscular" energy must pass through the radiation belt.

With a total of 6×10^{22} ergs, the maximum energy available for the earth's atmosphere cannot be more than 10^4 erg cm^{-2}. An energy supplied to the atmosphere of the order of 1 erg cm^{-2} sec^{-1}, which would be given by electrons of the order of 10 keV according to Krassovsky[44], will lead to the total energy of the radiation belt in about 3 hr. The hypothesis put forward by Krassovski[44] is very difficult to accept since the energy of the radiation should be renewed several times a day. Consequently, various suggestions of the normal heating of the upper atmosphere or of the creation of a heated atmosphere in a definite region of the auroral zone by the channelling of charged particles through the radiation belt cannot be accepted. Hence the deduction of Jastrow[35] that two atmospheres can exist simultaneously, i.e. a low latitude atmosphere with a temperature of the order of about 1000°K and one at a high latitude with a temperature of about 2000°K, has little plausibility.

An inflow of 2×10^{11} electrons cm^{-2} sec^{-1} leading to a heat source $Q = 4 \times 10^{16}$ cal cm^{-3} sec^{-1} at 300 km, as computed by Jastrow[35], corresponds to an injection into the thermosphere which is not observed even during auroras. Meredith[50] has given a specific example of a flux between 2 and 6×10^7 electrons cm^{-2} sec^{-1} of about 10 keV inside a diffuse aurora. Such an example shows that *normal* heating of the atmosphere by corpuscular radiation must be excluded. However, sporadic conditions corresponding to geomagnetic storms and leading to energies of 10^{22} ergs may lead to appreciable transient heating since it corresponds to 0·5 erg cm^{-2} sec^{-1} for the entire earth during one hour or about one-twentieth of the earth's surface during 24 hours.

Dessler[14] has put forward the view that the normal heating of the thermosphere is due to hydromagnetic heating. Against this view it can be stated that the diurnal variation of the temperature of the thermosphere cannot be explained by this process. But the increased orbital acceleration of satellites observed during geomagnetic storms at any latitude does not exclude a direct heating of the atmosphere outside the auroral zone. The hydromagnetic heating, therefore, must be kept among the hypotheses needed to explain a heating at very low latitudes during geomagnetic storms. A final decision cannot be reached until the variations of the atmospheric density are properly explored by using physical parameters such as the temperature and pressure in addition to the density and the scale height.

5. TRANSPORT OF HEAT BY CONDUCTION

5.1. *The equations of conduction*

The solar radiation heats the thermosphere at various altitudes, according to the values of the coefficients of absorption which range from 10^{-19} cm^2 to 10^{-16} cm^2. It is, therefore, appropriate to investigate the behavior of the atmosphere under the effect of the conduction of heat depending on the gradient of temperature.

The flux density of heat, E, can be written

$$E = -\lambda_c \operatorname{grad} T \tag{5.1}$$

where T is the temperature and λ_c is the thermal conductivity. If E is expressed in erg cm^{-2} sec^{-1}, λ_c is given in erg cm^{-1} sec^{-1} deg^{-1} and is related to the coefficient of viscosity, μ, by the equation[11]

$$\lambda_c = f\mu c_v \tag{5.2}$$

where c_v is the specific heat at constant volume and f represents a numerical value equal to about 2·5 for spherical molecules (monatomic gas) and can be equal to 1·9 for diatomic molecules.

The viscosity μ can be expressed as

$$\mu = \frac{5}{16} \frac{(\pi k m T)^{1/2}}{\pi \sigma^2} \tag{5.3}$$

where σ is the atomic radius, k Boltzmann's constant and m the atomic mass. This formula can be used in the high atmosphere[59, 60], and the values used are, for atomic hydrogen, $\sigma = 2{\cdot}0 \times 10^{-8}$ cm,

$$\mu(H) = 6{\cdot}8 \times 10^{-6} T^{1/2} \tag{5.4}$$

atomic oxygen, $\sigma = 2{\cdot}4 \times 10^{-8}$ cm,

$$\mu(\mathrm{O}) = 1{\cdot}9 \times 10^{-5} T^{1/2} \tag{5.5}$$

molecular nitrogen and oxygen, $\sigma = 3{\cdot}3 \times 10^{-8}$ cm

$$\mu(\mathrm{N_2, O_2}) = 1{\cdot}3 \times 10^{-5} T^{1/2} \tag{5.6}$$

In using equations (5.2) and (5.3) the thermal conductivity can be denoted by

$$\lambda_c = A T^{1/2} \tag{5.7}$$

where A is a constant, depending on the atmosphere constituents. The numerical value of this constant is (in erg cm^{-1} sec^{-1} deg$^{-3/2}$):

$$A(H) = 2{\cdot}1 \times 10^3 \tag{5.8}$$

$$A(\mathrm{O}) = 3{\cdot}6 \times 10^2 \tag{5.9}$$

$$A(\mathrm{O_2, N_2}) = 1{\cdot}8 \times 10^2 \tag{5.10}$$

Since λ_c is a function of the temperature, we introduce a new variable defined by

$$\theta = \int_{T_2}^{T} \frac{\lambda}{\lambda_2} dT \tag{5.11}$$

leading to the following relation, by using (5.7),

$$\theta = \frac{2}{3} \frac{T^{3/2} - T_2^{3/2}}{T_2^{1/2}} \tag{5.12}$$

Using (5.7) and (5.12) equation (5.1) can now be written as

$$E = -A T_2^{1/2} \operatorname{grad} \theta \tag{5.13}$$

The continuity equation is written as follows

$$\rho c_v \frac{\partial T}{\partial t} + \operatorname{div} E = P - L \tag{5.14}$$

in which P denotes the production of heat per unit of time, per unit of volume and L the loss of heat per unit of time and volume. Considering the heat capacity per unit of volume, ρc_v, where ρ is the density, equation (5.14) can be written as

$$\frac{\partial \theta}{\partial t} = \frac{A T^{1/2}}{\rho c_v} \nabla^2 \theta + (P - L) \frac{T^{1/2}}{\rho c_v T_2^{1/2}} \tag{5.15}$$

where the coefficient $AT^{1/2}/\rho c_v$ is the thermal diffusivity which, if n is the atomic concentration, can be written

$$\alpha(T, n) = A_1 T^{1/2}/n \tag{5.16}$$

where A_1 is a constant which has the following values (cm^{-1} sec^{-1} deg$^{-1/2}$)

$$A_1(H) = 1{\cdot}0 \times 10^{19} \tag{5.17}$$

$$A_1(\mathrm{O}) = 1{\cdot}75 \times 10^{18} \tag{5.18}$$

$$A_1(\mathrm{N_2, O_2}) = 5{\cdot}3 \times 10^{17} \tag{5.19}$$

As a result, the differential equation (5.15) for the conduction becomes:

$$\frac{\partial \theta}{\partial t} = \frac{A_1 T^{1/2}}{n} \nabla^2 \theta + \frac{A_1 T^{1/2}}{n A T_2^{1/2}} (P - L) \tag{5.20}$$

By using the variable T instead of introducing the temperature parameter θ, equation (5.20) would be non-linear. In fact the problem can always be studied by considering a condition at the limits $\theta_2 = 0$ for all fixed values of $T = T_2$.

If $\partial\theta/\partial t = 0$, equation (5.20) becomes Poisson's equation which can be applied to the steady state

$$\nabla^2 \theta + \frac{P - L}{A T_2^{1/2}} = 0 \tag{5.21}$$

If, on the other hand, there is no loss or production of heat at the centre of the volume being studied then (5.20) corresponds to the Laplace equation

$$\nabla^2\theta = 0 \tag{5.22}$$

When there is cooling by conduction with no production or loss of heat inside of the volume, equation (5.20) should be written

$$\frac{\partial\theta}{\partial t} = \frac{A_1 T^{1/2}}{n}\nabla^2\theta \tag{5.23}$$

in which, for ease of calculation, we can take a mean value of $A_1T^{1/2}$ and, thus, the thermal conduction depends, essentially, upon the value of the concentration n.

5.2. *Steady state with no source or loss of heat inside the volume*

In a sphere where the temperature is a function only of the radius r, the solution of (5.20) is

$$\theta = \theta_1 \frac{r-r_2}{r_1-r_2} \times \frac{r_1}{r} \tag{5.24}$$

if $\theta_2 = 0$ at a distance $r = r_2$ from the center of the sphere and θ_1 at $r = r_1$. According to (5.12), the distribution of the temperature becomes, (5.24),

$$(T^{3/2} - T_2^{3/2})/(T_1^{3/2} - T_2^{3/2}) = \frac{r-r_2}{r_1-r_2}\frac{r_1}{r} \tag{5.25}$$

The heat flow E_r is therefore (5.13) and (5.25),

$$E_r = \frac{r_2}{r}\frac{2}{3}A\ A\,(T^{3/2} - T_2^{3/2})/(r-r_2) \tag{5.26}$$

An immediate application is a night-time atmosphere where there is no heating by ultraviolet radiation at a sufficiently high altitude. In an atomic oxygen atmosphere if the heat flow has the following values at 500 km

0·1 0·2 0·5 erg cm^{-2} sec^{-1},

the corresponding values of the temperatures obtained at 700 km are

1250°K; $\Delta T = 350$ 1550°K; $\Delta T = 650$ 2300°K; $\Delta T = 1400$

1850°K; $\Delta T = 250$ 2100°K; $\Delta T = 500$ 2900°K; $\Delta T = 1300$

if the temperatures T_2 at 300 km are 900°K and 1600°K, respectively.

These numerical data show how the temperature would vary if an external heating were involved. However, since no direct determination of the temperature can be obtained from satellite data, it is necessary to consider (5.1) when the scale height H is used instead of T. With (5.9) and (5.10), the equations are

$$E\,(\mathrm{O}) = -0{\cdot}817 \times 10^{-3}\,(g/900)^{3/2} H^{1/2}\frac{dH}{dz} \tag{5.27}$$

for atomic oxygen, and

$$E\,(\mathrm{N_2, O_2}) = -0{\cdot}945 \times 10^{-3}\,(g/900)^{3/2} H^{1/2}\frac{dH}{dz} \tag{5.28}$$

for air, when $g = 900$ cm sec^{-1} is adopted for an altitude of 280 km.

If we consider the night-time data of Jacchia[33] between 600 and 700 km, it is clear that the energy which is required to maintain such a gradient according to (5.27) and (5.28), is

$$E\,(600\text{–}700\ \mathrm{km})_{\mathrm{night}} = (0{\cdot}28 \pm 0{\cdot}02)\ \mathrm{erg\ cm^{-2}\ sec^{-1}} \tag{5.29}$$

However, it has been shown (see Fig. 2) that at the *highest levels* it is possible to interpret the satellite observations concerned with atmospheric drag in terms of an isothermal atmosphere subject to diffusion.

As far as the daytime data are concerned, Jacchia[33] has deduced a more pronounced gradient which leads, in the same range of altitudes, to

$$E\,(600\text{–}700\ \mathrm{km})_{\mathrm{day}} \geqslant 1\ \mathrm{erg\ cm^{-2}\ sec^{-1}} \tag{5.30}$$

As a matter of fact, such a deduction corresponds to an increase of E with altitude, since near 600 km $E \simeq 1$ erg cm^{-2} sec^{-1} and near 700 km $E > 1{\cdot}3$ erg cm^{-2} sec^{-1}.

If at 700 km, a difference of about 1 erg cm^{-2} sec^{-1} distinguishes the difference between a sunlit and dark atmosphere (5.29) and (5.30), it would be necessary to assume that electromagnetic radiation can be absorbed at such high altitudes. The maximum possible total number of atoms in a vertical column, in using Jacchia's data[33], cannot be more than 8×10^{14} cm^{-2}.

Since any absorption cross-section cannot be more than 10^{-16} cm^2, the optical depth cannot be more than 0·08 and the ultra-violet energy needed would be more than 12·5 erg cm^{-2} sec^{-1}. Furthermore, since the absorption cross-section must be less than 10^{-16} cm^2, the ultra-violet radiation required should be more than 50 erg cm^{-2} sec^{-1} and such an energy will lead to temperature gradients below the unit optical depth of more than 500°K per km.

In fact, the gradients of temperature for $400°\text{K} \leqslant T \leqslant 1600°\text{K}$ are given by (E in erg cm^{-2} sec^{-1}):

$$\left(\frac{dT}{dz}\right)_{\text{km}} = (10 \pm 3)\, E \qquad (5.31)$$

for an atomic oxygen atmosphere, and

$$\left(\frac{dT}{dz}\right)_{\text{km}} = (20 \pm 6)\, E \qquad (5.32)$$

for an undissociated atmosphere. The scale height gradients, β, are under the same conditions, respectively

$$\beta\,(\text{O}) = (0{\cdot}51 \pm 0{\cdot}10)\, E \qquad (5.33)$$

and

$$\beta\,(\text{N}_2, \text{O}_2) = (0{\cdot}46 \pm 0{\cdot}10)\, E \qquad (5.34)$$

It is clear that E cannot be very different between day and night at highest altitudes since it would require a higher ultra-violet energy than is available from the sun. A corpuscular effect will not lead to a difference between day and night, and any external heating cannot explain such a difference. It must be concluded, therefore, that there is no physical process to be found to explain an increase of the temperature at such altitudes. An increase of the scale height gradient can be found only where the laws of diffusion can be employed, in fact at very high altitudes the gradient decreases and finally the scale height becomes constant.

Another aspect of a special behavior deduced from observational data is the gradient of the scale height (see Fig. 3) adopted by King-Hele and Walker[42] near 200 km. The gradient is negative between 210–220 km, $\beta = -0{\cdot}3$; it is positive between 200–210 km, $\beta = +0{\cdot}4$. Using formulas (5.27) and (5.28), a thin layer near 200 km should lead to an upward heat flow of (0·80 ± 0·06) erg cm^{-2} sec^{-1} and a downward heat flow of (1·15 ± 0·08) erg cm^{-2} sec^{-1}, i.e. a heat loss of about 2 erg cm^{-2} sec^{-1} for a layer of thickness less than 10 km. The total energy lost in one hour by such a layer would be more than its total kinetic energy. This shows that such a discontinuity cannot be a permanent feature in the thermosphere. We shall see later that such an artificial gradient would have a very short lifetime. In any case, it can be shown (see Fig. 3) that the vertical distribution of the density can be represented in an isothermal atmosphere subject to diffusion without such an anomaly in the scale height gradient. Therefore, the vertical distribution of the density near 200 km is not very sensitive to the exact value of the scale height gradient.

5.3. *Steady state with heating by ultra-violet radiation and loss by infra-red radiation*

In a steady state with one dimension the equation (5.21) is

$$\frac{\partial^2 \theta}{\partial z^2} + \frac{P - L}{A T_{\frac{1}{2}}^{1/2}} = 0 \qquad (5.35)$$

or

$$\frac{\partial E}{\partial z} + P = L \qquad (5.36)$$

Considering an atmosphere subject to a heating by several radiations, the production of heat P (cm^{-3} sec^{-1}) is

$$P = \Sigma n K E_{\text{uv}} \exp\left(-\int_z^{\infty} nK dz\right) dz = \Sigma E_{\text{uv}} e^{-\tau} d\tau \qquad (5.37)$$

in which K is the appropriate absorption cross-section of the radiation E_{uv} of wavelength λ by atoms of concentration n. τ is the optical depth defined by

$$d\tau = -nK dz \qquad (5.38)$$

leading to

$$\tau = \int_z^{\infty} nK dz \qquad (5.39)$$

In the thermosphere, the principal infra-red radiator is atomic oxygen, according to Bates's process[3]

$$\text{O}\,(^3P_1) \longrightarrow \text{O}\,(^3P_2) + h\nu\ (\lambda = 63\mu) \qquad (5.40)$$

since molecular nitrogen and oxygen have no dipole. The loss of heat (cm^{-3} sec^{-1}) is therefore (in erg cm^{-3} sec^{-1})

$$L = Rn(O) = n(O) \, (1{\cdot}68 \times 10^{-18} e^{-228/T}) / (1 + 0{\cdot}6 \times e^{-228/T} + 0{\cdot}2 \times e^{-325{\cdot}3/T}) \tag{5.41}$$

leading to

$$L(E\text{-layer}) = (5 \pm 1) \times 10^{-19} n(O) \text{ erg cm}^{-3} \text{ sec}^{-1} \tag{5.42a}$$

$$L(F\text{-layer}) = (8 \pm 0{\cdot}5) \times 10^{-19} n(O) \text{ erg cm}^{-3} \text{ sec}^{-1} \tag{5.42b}$$

Using (5.37) and (5.40), integration of (5.36) leads to

$$E = E_{\infty} + E_{uv}(1 - e^{-\tau}) + \bar{R} n(O) H(O) (g/\bar{g}) \tag{5.43}$$

in which E_{∞} denotes the external heat available for the layer, and $\bar{R}$ and $\bar{g}$ are mean values.

Since the absorption cross-section in the ultra-violet range is $K \geqslant 10^{-17}$ cm^2 and $R < 10^{-18}$ erg sec^{-1},

$$E_{uv}(1 - e^{-\tau}) > \bar{R} n(O) H(O) \tag{5.44}$$

and above the unit optical depth, it is clear that the radiation loss is also negligible. A loss of heat of the order of 1 erg cm^{-2} sec^{-1} requires 2×10^{18} cm^{-2} atoms of oxygen, i.e. a layer of 10 km with concentration of 2×10^{12} atoms cm^{-3} which corresponds to an altitude of the order of 100 km. It can be concluded that the loss of heat at 63μ occurs mainly inside the E-layer, where the radiation is certainly of the order of 1 erg cm^{-2} sec^{-1}. In the F-layer the total loss of a vertical column would be of the order of 0·1 erg cm^{-2} sec^{-1}: thus the loss of heat by radiation may be neglected in a sunlit atmosphere above the E-layer when the ultra-violet energy available is at least 1 erg cm^{-2} sec^{-1}. On the other hand, any ultra-violet energy of the order of 0·1 erg cm^{-2} sec^{-1} absorbed above the level of the $F1$ peak is of the order of the heat loss by atomic oxygen since $K \simeq 10R$.

The integration of (5.43) with the use of (5.27) and (5.28) may lead to a determination of the possible increases of the scale height (H_{∞}) up to the isothermal layer. The result can be written as follows (neglecting the variation of g):

$$H_{\infty}^{1/2} = H_0^{1/2} + \frac{E_{\infty}}{2A} \log \tau_0/\tau + \frac{E_{uv}}{2A} [\log \tau_0 - Ei(-\tau_0) + 0{\cdot}57722] - \frac{\bar{R}}{2A} [n_0(O) H_0(O) - n(O) H(O)] \tag{5.45}$$

where A is the constant of (5.27) or (5.28).

Adopting $\tau_0 = 1$, near the absorption peak, the effect of the ultra-violet radiation on the scale height can be shown by the following examples:

If $H_0 = 40$ km ($\tau = 1$), H_{∞} has the values (± 7 per cent)

H =	60 km	84 km	113 km	145 km
when				
E_{uv} =	1	2	3	4 erg cm^{-2} sec^{-1}

If $H_0 = 50$ km ($\tau = 1$), H_{∞} reaches the values

H =	72 km	99 km	129 km	164 km
when				
E_{uv} =	1	2	3	4 erg cm^{-2} sec^{-1}

In other words, the scale height varies by a factor of 2 between the absorption peak and the beginning of the isothermal layer when the ultra-violet energy available for heating is of the order of 2 erg cm^{-2} sec^{-1}. Since diffusion is also involved, it is evident that scale heights greater than 100 km can be reached if the ultra-violet energy is of the order of 1 erg cm^{-2} sec^{-1}.

Evidence favouring an important gradient of temperature in a sunlit atmosphere is shown by the occurrence of pronounced diurnal variations of the density. An exact vertical distribution of the temperature cannot be obtained since the solar spectrum in the ultra-violet must be known in all the details and also the exact, varying, ratio N_2/O. However, the diminution in the temperature gradient should be indicated by a law of the form

$$\frac{dT}{dz} \propto \Sigma E_{uv}(1 - e^{-\tau}) \tag{5.46}$$

for an overhead sun.

It is possible to obtain some idea of the conditions which are required to reach the observed densities at 200 km and scale heights deduced from (5.45). If we assume, for example, that below the specific level of 150 km (see equation 3.4), the effect of convection is such that the energy E follows the following law

$$E = E_{uv}\left(\frac{H}{H_h}\right)^{1/2} \propto \frac{v}{v_h} \qquad (5.47)$$

where H_h corresponds to 150 km, $H < H_h$ and v is the kinetic energy. The scale height gradient is, according to (5.28),

$$\beta = E_h / A H_h^{1/2} = 1{\cdot}058 \times 10^3 \, (E_h / H_h^{1/2}) \qquad (5.48)$$

As was said before (see equation 3.4), the density remains almost constant at 150 km for any gradient $0{\cdot}50 \leqslant \beta \leqslant 1{\cdot}50$. The energy necessary for such gradients varies according to (5.48) from 0·7 erg cm^{-2} sec^{-1} to 3·3 erg cm^{-2} sec^{-1} (see Fig. 12). The solar energy which must be available for the heating cannot be determined if the absorption cross-section is not known. Let us assume for example a mean value of 2×10^{-17} cm^2 which corresponds to the most important part of the ultra-violet spectrum. In such a case, the following ultra-violet energies

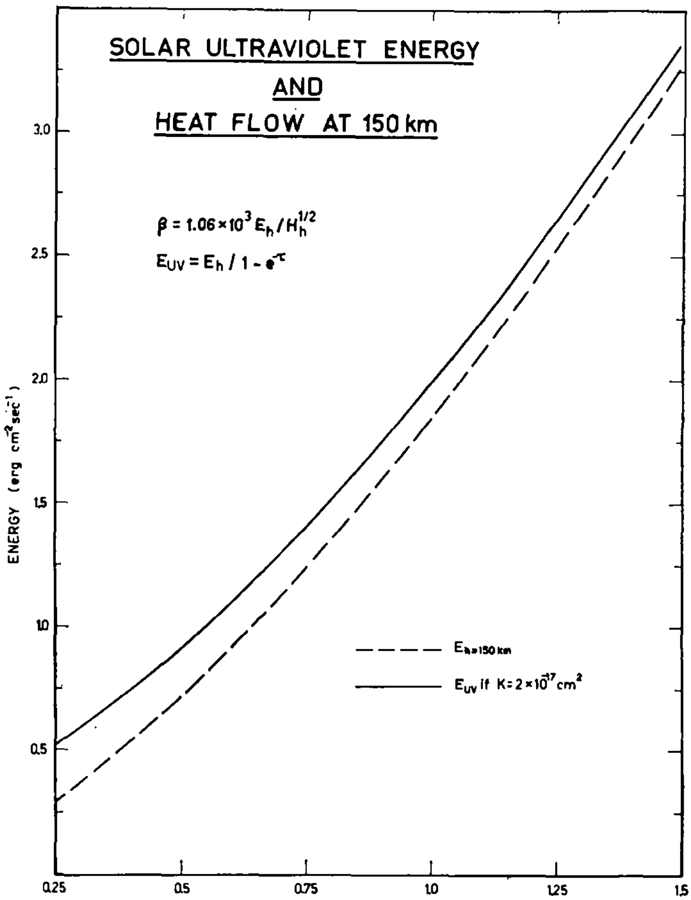

Fig. 12. Heat flow and scale height with constant scale height gradient. The effect of thermal conductivity at 150 km is shown by a curve relating the scale height gradient and the energy necessary to maintain a certain gradient. Assuming an absorption cross-section of the order of 2×10^{-17} cm^2 the ultra-violet energy necessary at the top of earth's atmosphere E_{uv} is deduced.

E_{uv} at the top of the earth's atmosphere would be necessary:

β	0·5	0·75	1·00	1·25	1·50	
E_{uv}	0·9	1·4	2·0	2·7	3·4	erg cm^{-2} sec^{-1}

and the corresponding temperatures at 150 km (see Fig. 13) would be:

T	725	960	1190	1425	1650°K

Such scale height gradients and temperatures would correspond to real parameters since the total kinetic energies which are involved in the atmosphere above 150 km are between 10^4 erg cm^{-2} and 6×10^4 erg cm^{-2} corresponding to E_{uv} from 0·9 erg cm^{-2} sec^{-1} to 3·4 erg cm^{-2} sec^{-1}, respectively. In other words, the solar energy which is used during one day of 12 hr is of the same order as the kinetic energy of the vertical column above 150 km. It is, consequently, certain that the variations in the thermosphere above 100 km can be associated with the variations of the solar energy and that the variations of the temperature and its gradient are closely associated with the absorption processes below 200 km. The temperature at the thermopause depends strongly on its gradient below 200 km. But the level of the thermopause must be subject to a diurnal variation according to the laws of cooling after sunset.

5.4. *Cooling of the atmosphere after sunset*

The application of equation (5.23), after sunset, in the following form

$$\frac{\partial \theta}{\partial t} = (A_1 T^{1/2}/n) \times (\partial^2 \theta / \partial z^2) \qquad (5.47)$$

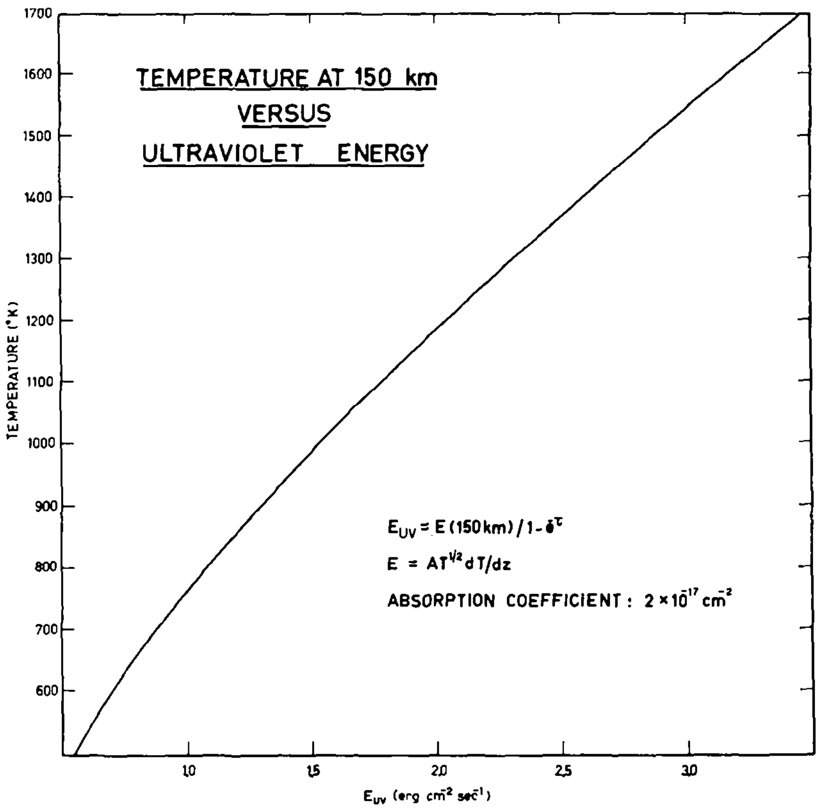

Fig. 13. Temperature at 150 km and ultra-violet energy available at the top of the earth's atmosphere. From the abscissae representing the ultra-violet energy, may be deduced the temperature at 150 km if the absorption cross-section is assumed.

corresponding to an initial atmosphere with a certain vertical gradient leads to the following results.

Let us consider, as an example, an atmosphere* giving approximately the vertical distribution of the observed density and corresponding to a heat flow of about 0·3 erg cm^{-2} sec^{-1} through atomic oxygen. If the variation is very small at 200 km, it is of the order of a factor of 10 near 650 km in about 12 hr. There is a clear indication that the atmosphere becomes rapidly isothermal at the highest altitudes and the isothermy extends with time to lower levels. It must be pointed out that $\rho H^{1/2}$ at highest altitudes decreases rapidly during the first hours, and it is interesting to note that the variation is very small between 12 and 24 hr compared with the variation during the first 12 hr.

In fact, the tendency to isothermy is very rapid when the density is less than 5×10^{-15} g cm^{-3} or concentrations less than 10^8 cm^{-3}. Equation (5.47) shows that if a temperature gradient corresponding to a heat flow of the order of 0·3 erg cm^{-2} sec^{-1} exists in an atomic oxygen atmosphere, it disappears in less than 30 min between 450 km and 750 km. Such a result shows that it is not possible to consider a dark atmosphere with a temperature gradient above 300–350 km. Moreover, the temperature of the isothermal atmosphere decreases in a continuous way following a decrease in the temperature gradient between 200 km and 300 km.

The absolute value of the decrease, being obviously a function of the initial gradient, the behavior of the variation depends upon the vertical distribution of the ultra-violet radiation absorbed in the sunlit atmosphere. Nevertheless, it is clear that the variation of the temperature of the isothermal layer between $t=30$ min and $t=2$ hr is of the same order as that between $t=2$ hr and $t=12$ hr. In other words, after a rapid tendency towards isothermy at highest altitudes, the temperature of the isothermal layer decreases according to the rapidity with which the isothermal layer extends downwards and the variation of the temperature below 200 km.

As a consequence, the heat transport by conduction associated with the gradient of temperature that results from the absorption of solar ultra-violet radiation explains the diurnal variation of the density of the thermosphere deduced from the variation of the acceleration of the satellites. We must also conclude that the diurnal variation of the temperature of the isothermal region is of the order of 500°K and that the temperature gradient is certainly subject to a variation down to 150 km.

Lastly, we must realize that the heating during magnetic storms will have an effect during a time of the order of one day since the effect of the solar electromagnetic radiation leads to a strong diurnal variation. Consequently, any sporadic effect, if energetic enough, will modify the atmospheric structure only during its own lifetime and its role will be more or less effective if it occurs in a dark or sunlit atmosphere, respectively.

5.5. *Times of conduction*

The application of equation (5.47) to the problem of the cooling of the atmosphere after sunset indicates that the time required for the transport of heat varies greatly with altitude and distance. Actually, the time of conduction is proportional to the concentration, n, and to the square of the distance.

In order to provide order of magnitudes for times of conduction, we apply equation (5.47) to very simple examples. Let us consider two infinite regions of the atmosphere where the conditions are such that the temperatures are initially T_1 $(x \leqslant 0)$ and T_2 $(x > 0)$. The redistribution of the temperature (isothermy) is given by the solution of (5.47):

$$\theta/\theta_1 = 1 - \psi(\mu) \qquad (5.48)$$

when T_1 is kept constant $(T \longrightarrow T_1)$ and $x > 0$ or

$$\theta/\theta_1 = \tfrac{1}{2}[1 + \psi(\mu)] \qquad (5.49)$$

if $T \longrightarrow \frac{1}{2}(T_1 + T_2)$ and $-\infty < x < +\infty$. In (5.48) and (5.49),

$$\psi(\mu) = \frac{2}{\sqrt{2\pi}} \int_0^{\mu} e^{-y^2} dy \qquad (5.50)$$

* Such an atmosphere does not represent real conditions and has been chosen as a working model.

and

$$\mu = \frac{x}{2}\left(\frac{n}{A_1 T^{1/2} t}\right) \tag{5.51}$$

where t denotes the time; A_1 is given by (5.18) or (5.19).

By using (5.12), we can write the relation between the temperatures if we adopt a definite value for $\psi(\mu)$. Since $\psi(\mu)=0{\cdot}2$ leads to conditions near to isothermy, such a value is adopted and introduced in (5·48) and (5.49). With (5.48), the solutions are:

If $T_2/T_1=0{\cdot}5$, $T/T_1=0{\cdot}91$, ..., if $T_2/T_1=0{\cdot}9$, $T/T_1=0{\cdot}98$, and with (5.49),
if $T_2/T_1=0{\cdot}5$, $T/T_1=0{\cdot}80$, ..., if $T_2/T_1=0{\cdot}9$, $T/T_1=0{\cdot}95$.

It is, therefore, possible to define a time of conduction related to the concentration (see 5.51) and the distance x. Using the constants from the formulas (5.17), (5.18) and (5.19), we obtain the times of conduction t_{sec} in seconds for atomic hydrogen, atomic oxygen and air

$$t_{sec}(H) = 7{\cdot}72 \times 10^{-9} n(H)\, x_{km}^2 T^{-1/2} \tag{5.52}$$

$$t_{sec}(O) = 4{\cdot}41 \times 10^{-8} n(O)\, x_{km}^2 T^{-1/2} \tag{5.53}$$

$$t_{sec}(N_2, O_2) = 1{\cdot}46 \times 10^{-7} n(O_2, N_2)\, x_{km}^2 T^{-1/2} \tag{5.54}$$

For a temperature range of 900°K to 2500°K, i.e. for $\bar{T}=(40 \pm 10)^2$ °K, the following times of conduction in seconds for distances x expressed in km and concentrations in cm^{-3} are obtained:

$$t(H) = (2 \pm 0{\cdot}5) \times 10^{-10} x_{km}^2 n(H) \tag{5.55}$$

$$t(O) = (1{\cdot}2 \pm 0{\cdot}3) \times 10^{-9} x_{km}^2 n(O) \tag{5.56}$$

$$t(O_2, N_2) = (3{\cdot}8 \pm 10) \times 10^{-9} x_{km}^2 n(O_2, N_2) \tag{5.57}$$

If a horizontal discontinuity exists in the horizontal temperature distribution it cannot be maintained at the highest altitudes. For example, the isothermy should be reached after about 4 hr at a distance of 1000 km in an atmosphere (600–700 km) with concentration of 10^7 oxygen atoms cm^{-3}. This means that a difference of temperature in latitude can only be maintained during a very short time at high altitudes. Furthermore, a seasonal effect cannot be related to differences of temperatures at high altitudes. All conditions are related to the diurnal variations which are important according to the results of the analysis made in Section 5.4.

If we consider that horizontal layers of a certain thickness x occur with different temperatures T_1 and T_2, it is possible to consider the time of conduction which ought to be taken into account to reach isothermy. For example, King-Hele and Walker[42] in their atmospheric model have introduced a peak in the scale height at 210 km ($H=47$ km) and a minimum at 250 km ($H=40$ km). For a molecular nitrogen atmosphere, equation (5.47) shows that

$$\theta/\theta_1 = \tfrac{1}{2}\psi(\mu) \tag{5.58}$$

where

$$\mu^2 = a^2 n / A_1 T^{1/2} t \tag{5.59}$$

if a is the thickness of the layer with initial temperature T_1 ($x < a$) and initial temperature T_2 for $-a < x < a$.

Therefore the isothermy is reached ($\mu=0{\cdot}2$ and $900°\text{K} \leqslant \bar{T} \leqslant 1600°\text{K}$) when the time of conduction t_{sec} is

$$t_{sec} = 1{\cdot}7 \times 10^{-8} a_{km}^2 n \tag{5.60}$$

Since the concentration is about 3×10^9 cm^{-3} near 250 km and about four times more near 200 km, the temperature discontinuity will disappear in about 2×10^4 sec between 200 km and 280 km. These results, related to those obtained for the horizontal heat transport, clearly show that discontinuities in the thermospheric temperatures cannot be permanent.

6. THERMOSPHERIC CONDITIONS AND ATMOSPHERIC MODELS

Heat conduction times which are appropriate to the thermosphere are between 12 and 24 hr since the diurnal variation resulting from the heating by electromagnetic radiation is the more pronounced effect. Any gradient of temperature which is not maintained by an external heat flow has a lifetime of less than one night above 350 km, and in a dark atmosphere such an altitude can be considered as being in the neighborhood of the thermopause level. In a sunlit atmosphere its altitude will depend on the solar energy available and the variable temperature of the isothermal region will be a function of the temperature gradient between 100 km and 200 km. As we have shown previously a change of the gradient below 200 km does not greatly change the atmospheric density at 200 km, but the atmosphere above 300 km is strongly affected. However, a general effect associated to the solar activity (27-day period and sunspot cycle), producing a modification of boundary conditions in the *E*-layer will affect the atmosphere near 200 km with amplified effects at increasing altitudes.

A "corpuscular effect" will also have a detectable lifetime of the order of one day since it will be involved in the effect of the diurnal variation. On the other hand, such an effect will be much easier to detect in a dark atmosphere when the temperature reaches its normal minimum.

The diurnal variation has another important effect on the altitude of the conventional exosphere, because differences of several hundreds of km make it impossible to determine relations connected with the outer atmosphere on a permanent basis. It is, therefore, necessary to take into account diurnal and solar variations modifying simultaneously the values of the density and the temperature. It should be understood that a diurnal variation of diffusion should exist up to the highest altitudes and that the ratio N_2/O must vary between day and night. On the other hand, the upper part of the ionosphere should be subject to very important diurnal and solar effects. Since the recombination in the *F*-layer depends on reactions related to the temperature, important diurnal variations must occur in the rate coefficients. Above the *F2*-peak there must be a diurnal variation in the scale height associated with the vertical distribution of electrons subject to diffusion.

If it is relatively easy to represent atmospheric conditions in a dark atmosphere where the thermopause is at a relatively low altitude, it is very difficult to find an adequate picture of the atmosphere under sunlit conditions until the solar spectrum is known exactly.

An atmospheric model based solely on a heat flow by conduction from outside the atmosphere[57] is an extreme example which cannot be associated with a diurnal variation of the density. Such a heating could be easily explained by the Chapman[8, 9] process, suggesting that the solar corona extends to the limits of the terrestrial atmosphere. However, it must be considered as a very small fraction in relation to the heating due to the ultra-violet radiation and, in fact, it is negligible in comparison. In the same way, corpuscular and hydromagnetic effects are not important under normal conditions, but disturbed conditions associated with magnetic storms, must lead to transient atmospheric conditions in which the temperature of the isothermal layer is higher than that in the night-time atmosphere.

If it is clear that the scale height increases with height and that its gradient may reach a peak at a certain altitude, it is also evident that heat conduction does not permit having a temperature gradient increasing with height; even when the temperature increases up to the highest altitudes. For this reason, the atmospheric models such as those of Mikhnevich *et al.*[52], Champion and Minzner[7], Kallmann[36], etc., do not represent real physical conditions.

In the Mikhnevich model the temperature gradient increases with the altitude by $1°K\ km^{-1}$ at 250 km, 2° at 300 km, 3° at 350 km, 4° at 425 km, 6° at 450 km and 7° at 500 km. Such an increase of the temperature gradient would represent a downward heat flow at 500 km at least ten times greater than the heat flow at 250 km. No permanent physical process could explain such a result.

In the Champion and Minzner model, the temperature difference by steps of 100 km from 200 km to 700 km changes successively from 19°K to 57°K, 96°K, 115°K and 121°K. Such an increase of the temperature gradient with height associated with an isothermy between 210 and 260 km would lead also to a downward heat flow; maximum at the highest levels and disappearing near 250 km. Hence the high temperatures near 700 km cannot result from the vertical distribution deduced by Champion and Minzner[7].

In summary, all atmospheric models involving an increase of the gradient of temperature with height cannot be accepted. Such models lead to the ratio of heat flows E and E_0,

$$\frac{E}{E_0}=\frac{AT^{1/2}dT/dz}{A_0T_0^{1/2}dT_0/dz} \tag{5.61}$$

in which E is for a height z and E_0 is for a height $z=0$. Since $AT^{1/2}>A_0T_0^{1/2}$ and $E\leqslant E_0$, if there is no radiation loss, dT/dz must be less than dT_0/dz. On the other hand, since the conduction of heat is a very rapid process at highest altitudes there is no way to avoid conclusions obtained from (5.61).

Considering now that the tendency of the atmosphere after sunset is to attain isothermy, a night-time model of the atmosphere must be represented at the highest levels by a quasi-isothermal atmosphere in which the constituents are subject to diffusion. In Section 2.1 (see Fig. 2), it has been shown that an isothermal atmosphere subject to diffusion is not far from a quasi-isothermal atmosphere agreeing with night-time satellite data. Furthermore, in Section 3, it has been found that the density (expression 3.4) at 150 km is not sensitive to the temperature gradient when constant boundary conditions are assumed at 120 km. For these two reasons it can be assumed that atmospheric models for night-time conditions at the highest altitudes must be considered in such a way that the temperature is nearly constant and that diffusion affects the atmospheric scale height.

In such circumstances, an important parameter is the ratio of the densities of molecules and atoms. We start from conditions given by (3.4), i.e. at 150 km,

total density: $\rho=1{\cdot}5\times10^{-12}\,\mathrm{g\,cm^{-3}}$ (5·62)

temperature: $725°\mathrm{K}\leqslant T\leqslant1650°\mathrm{K}$ (5.63)

concentrations: $n(N_2)=2{\cdot}6\times10^{10}\,\mathrm{cm^{-3}}$ (5.64)

$n(O)=3{\cdot}3\times10^{9}\,\mathrm{cm^{-3}}$

$n(O_2)=5{\cdot}0\times10^{9}\,\mathrm{cm^{-3}}$

leading to the approximate ratios

$$n(M):n(N_2):n(O_2):n(O)=1:0{\cdot}76:0{\cdot}14:0{\cdot}10 \tag{5.65}$$

The effect of diffusion is to lead, at a certain altitude, to the following relation between the densities

$$\rho(O_2, N_2)=\rho(O) \tag{5.66}$$

corresponding to a certain total density ρ. Computation shows that for the conditions (5.65), (5.66) is obtained where the total density has a definite value, i.e.

$$\rho=(4{\cdot}45\pm0{\cdot}05)\times10^{-15}\,\mathrm{g\,cm^{-3}} \tag{5·67}$$

This important relation (5.67) leads to the immediate conclusion that the temperature of the isothermal atmosphere can be found when the altitude of $\rho(O_2, N_2)=\rho(O)$ is known.

The following results leading to a "thermo-isobaric relation" have been obtained:

Table 1.

Altitude in km of $\rho(O_2, N_2)=\rho(O)$	Temperature (°K)
300	725
350	958
400	1191
450	1424
500	1657

Fig. 14 shows the thermo-isobaric relation, i.e. the relationship between the temperature of the isothermal layer and the altitude of the constant density given by (5.67) corresponding to the same densities for atoms and molecules. The concentrations are approximately as follows:

$$n(N_2)=4{\cdot}4\times10^{7}\,\mathrm{cm^{-3}};\quad n(O_2)=3{\cdot}3\times10^{6}\,\mathrm{cm^{-3}};\quad n(O)=8{\cdot}6\times10^{7}\,\mathrm{cm^{-3}} \tag{5.68}$$

The vertical distribution of the scale height is shown in Fig. 15 for various isothermal atmospheres. An increase of H from 50 km at 250 km to 100 km at 1000 km is given for a temperature of the order of 1400°K. Such an increase results only from the decrease of the molecular mass with altitude. It can also be seen that the fluctuations of the scale height deduced by King-Hele and Walker[42] are anomalous variations of the scale heights in an isothermal atmosphere around 1200°K; not too different from the temperature $T=1250°\mathrm{K}$ deduced (Fig. 14) from the thermo-isobaric relation.

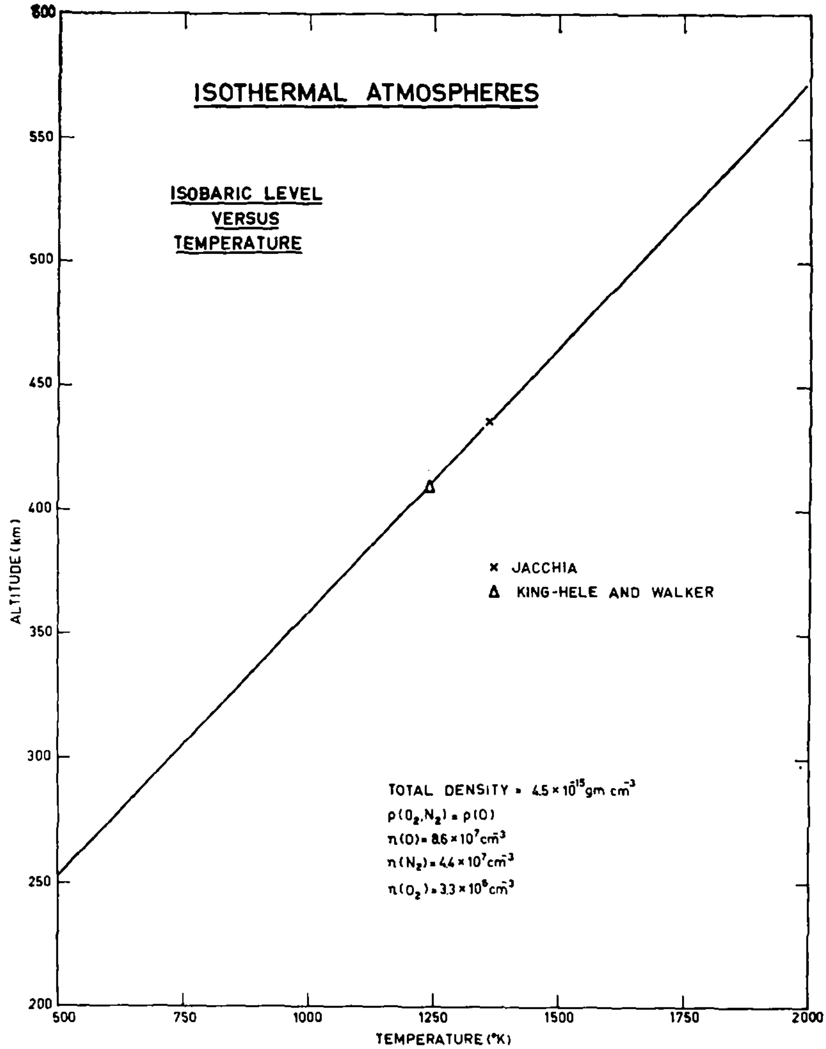

Fig. 14. Temperature–altitude relation for a total density $\rho = 4{\cdot}5 \times 10^{-15}$ gm cm^{-3} where ρ (O_2, N_2) = ρ (O) in an isothermal atmosphere. The knowledge of the altitude of the isobaric level leads to a determination of the temperature and therefore of the vertical distribution of the density.

Fig. 16 shows the vertical distribution of the density for which the constant density defined by (5.67) corresponds to the equality of the density of molecules and atoms. As was shown before (Fig. 2) the curves leading to (5.66) at 400 km and 450 km are not far from the night-time densities deduced by King-Hele and Walker[42] and Jacchia[33], respectively.

It is clear that the preceding determination of night-time conditions of the isothermal layer does not give a complete answer since it depends on boundary conditions for diffusion and temperature. The atmospheric model is consistent in the use of all physical parameters; however, it represents only a guide to the study of atmospheric behavior. In order to show how small

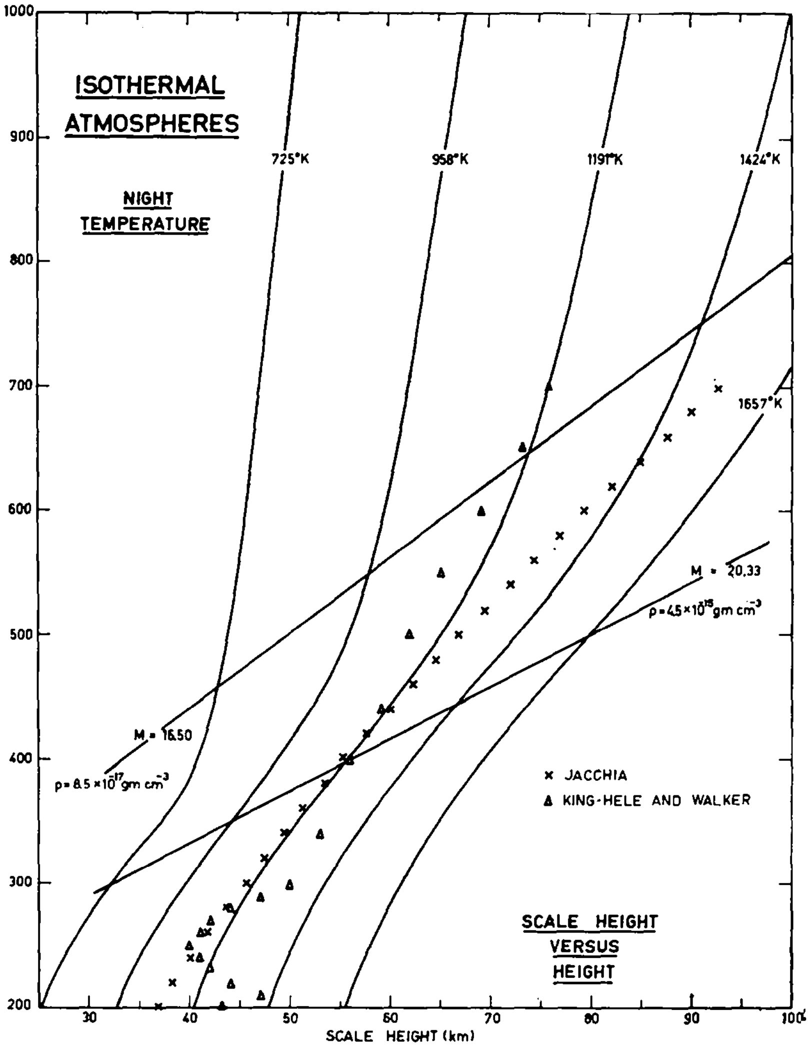

Fig. 15. Scale height-altitude relations for various isothermal atmospheres. Scale height values deduced by Jacchia'[38] and King-Hele and Walker[42] are shown with their different vertical distribution.

differences in physical conditions can modify the conclusions, instead of (5.66), we assume

$$\rho(O_2, O) = \rho(N_2) \qquad (5.69)$$

corresponding again for all temperatures to a constant density. The expression (5·69) leads to a different distribution of the density, even if the concentration of atomic oxygen, $n(O) = 8{\cdot}5 \times 10^7$ cm^{-3}, remains the same at the thermo-isobaric level. The vertical distribution of density for various temperatures are shown in Fig. 16 where it is interesting to compare the results represented by the dotted curves with the data shown by continuous curves.

Computation shows that the condition (5.69) is obtained where the total density has the definite value

$$\rho = (4{\cdot}75 \pm 0{\cdot}25) \times 10^{-15} \text{ g cm}^{-3} \qquad (5.70)$$

This thermo-isobaric relation leads to the results of Table 2.

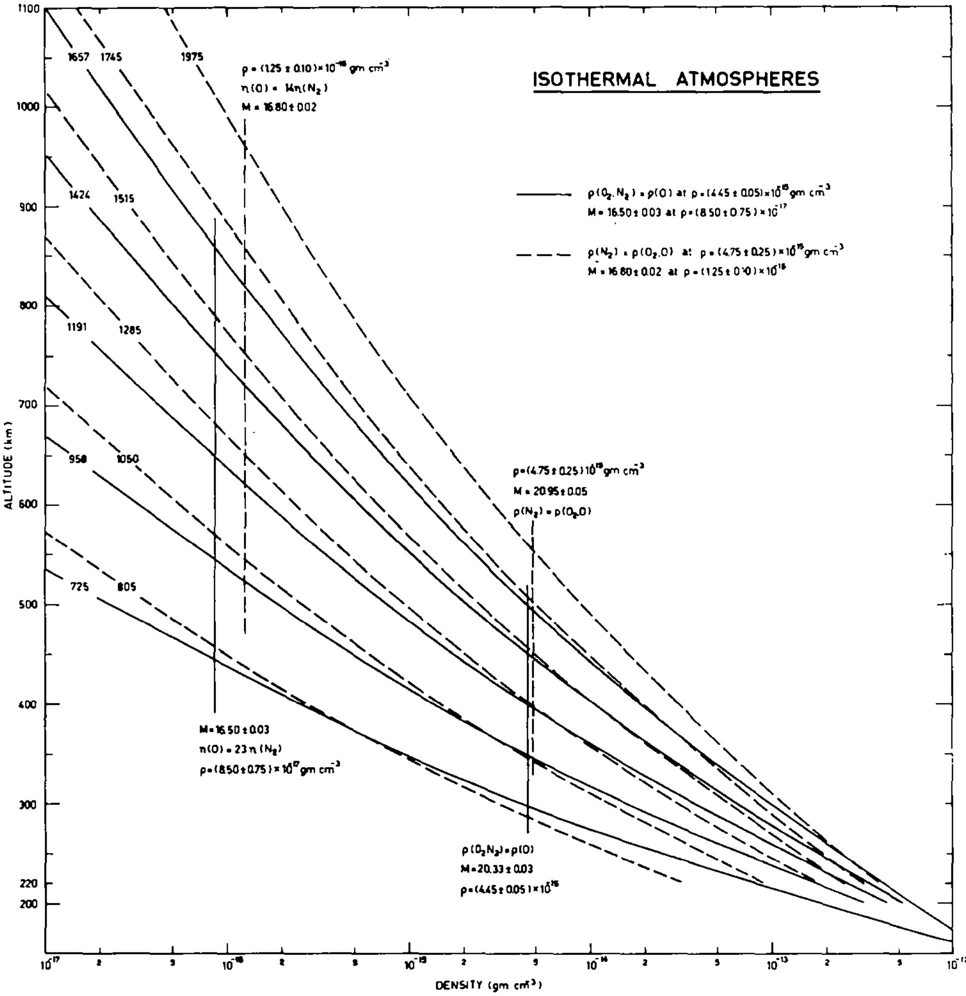

Fig. 16. Densities in isothermal atmospheres. The absolute values of density are determined by conditions ρ (O_2, N_2)$=\rho$ (O) at an isobaric level $\rho=4{\cdot}5\times10^{-15}$ g cm^{-3} and ρ (O_2, O)$=$ ρ (N_2) at an isobaric level $4{\cdot}75\times10^{-15}$ g cm^{-3} leading to densities at 220 km corresponding to observed values and to remarkable differences at highest altitudes.

Table 2.

Altitude in km of $\rho(N_2)=\rho(O_2, O)$	Temperature (°K)
350	1050
400	1285
450	1515
500	1745
550	1975

Table 3(a)

Density at 220 km 10^{-13} g cm^{-3}	Isothermal Atmosphere (°K) T_1	Isothermal Atmosphere (°K) T_2
(1) 0·32	—	805
(2) 0·85±0·5	725	1050
(3) 1·7 ±0·1	958	1285
(4) 2·5 ±0·1	1190	1515
(5) 3·2 ±0·1	1424	1745
(6) 3·9 ±0·1	1657	1975

The main conclusions are given in Tables 3(a) and 3(b).

Table 3(b)

	$\rho_1 = 4{\cdot}45 \times 10^{-15}$ Altitude (km)	$\rho_2 = 4{\cdot}75 \times 10^{-15}$ Altitude (km)	$\rho_1 = 8{\cdot}5 \times 10^{-17}$ Altitude (km)	$\rho_2 = 1{\cdot}25 \times 10^{-16}$ Altitude (km)
(1)		300	—	450
(2)	300	350	450	550
(3)	350	400	550	650
(4)	400	450	650	750
(5)	450	500	750	850
(6)	500	550	850	950
	$M = 20{\cdot}33 \pm 0{\cdot}03$	$M = 20{\cdot}95 \pm 0{\cdot}05$	$M = 16{\cdot}50 \pm 0{\cdot}03$	$M = 16{\cdot}80 \pm 0{\cdot}02$

The main conclusions are given in Tables 3(a) and 3(b).

Starting from thermo-isobaric levels leading to $4{\cdot}5 \times 10^{-15}$ g cm^{-3} and $4{\cdot}75 \times 10^{-15}$ g cm^{-3}, corresponding to $\rho(O_2, N_2) = \rho(O)$ and $\rho(O_2, O) = \rho(N_2)$, respectively, i.e. mean molecular masses $M = 20{\cdot}33$ and $M = 20{\cdot}95$, the same altitudes are obtained if the temperatures difference is of the order of 100°K. An altitude difference of 50 km corresponds to isothermal atmospheres for which the temperatures differ from each other by about 300°K. In such a case they lead to the same density at 220 km; at the highest altitudes, a difference of more than 100 km is required to reach the same density.

Such large differences show that the structure of the high atmosphere is very sensitive to diffusion and there is a possibility of finding physical conditions which can represent the conditions of a dark atmosphere. From the preceding table and from satellite data, the following data may be taken as a guide (see preceding table): $\rho(220 \text{ km}) = (2{\cdot}5 \pm 0{\cdot}7) \times 10^{-13}$ for $960°\text{K} \leqslant T \leqslant 1425°\text{K}$ leading to $\rho = 4{\cdot}5 \times 10^{-15}$ g cm^{-3} for $350 \text{ km} \leqslant z \leqslant 450$ km and $\rho = 8{\cdot}5 \times 10^{-17}$ g cm^{-3} for $550 \text{ km} \leqslant z \leqslant 750$ km. The altitudes for $\rho = 4{\cdot}5 \times 10^{-15}$ g cm^{-3} and $\rho = 8{\cdot}5 \times 10^{-17}$ g cm^{-3} are related directly to the temperature. Furthermore, the concentrations are given by (5.68) where $\rho = 4{\cdot}5 \times 10^{-15}$ g cm^{-3} and $n(O) = 23n(N_2)$ where $\rho = 8{\cdot}5 \times 10^{-15}$ g cm^{-3}. Finally, the mean molecular masses are approximately $M = 20{\cdot}33$ at $\rho = 4{\cdot}5 \times 10^{-15}$ g cm^{-3} and $M = 16{\cdot}50$ at $\rho = 8{\cdot}5 \times 10^{-17}$ g cm^{-3}.

These various values will serve as a guide in the analysis of night-time conditions in the ionosphere, of airglow and auroras keeping in mind that in a normal dark atmosphere during maximum sunspot conditions the temperature of the isothermal atmosphere may reach values between 1200°K and 1400°K. However, values of the order of 1000°K can be easily accepted.

The problem of the sunlit atmospheres must be studied under various conditions depending on the ultra-violet solar radiation available and on the vertical distribution of its absorption.

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