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Total electron content monitoring using triple frequency GNSS data: A three-step approach

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ABSTRACT

Global Navigation Satellite Systems (GNSS) are very useful tools to study the ionosphere. Nevertheless, the precision of the usual dual frequency total electron content (TEC) monitoring technique is affected by code delays (hardware and multipath), and is therefore limited. This paper introduces a TEC monitoring technique based on triple frequency GPS and Galileo measurements. The three steps of this technique are validated on triple frequency simulated data. In fact, as it is not affected by code delays, the precision of the reconstructed TEC is improved in regards with the dual frequency technique.

1. Introduction

The Global Positioning System (GPS) has become a very useful tool to study the ionosphere. In fact, the total electron content (TEC) can be determined by using the so-called dual frequency geometric-free (GF) combinations (Warnant and Pottiaux, 2000). In that method, TEC is computed from the GF phase combination, which is precise but ambiguous. Therefore, the use of the GF code combination is required for the ambiguity resolution process, so that code multipath delays and differential satellite and receiver code hardware delays affect the precision of the reconstructed TEC (Ciraolo et al., 2007). Warnant and Pottiaux (2000) have shown that the usual mid-latitude precision is limited to 2–3 TECU.

Triple frequency Global Navigation Satellite Systems (GNSS)—both GPS and Galileo—will be operational in the next few years. The first Galileo test satellite, named Giove-A, is already in orbit since December 2005. The availability of a third frequency allows to develop several innovative techniques. We can cite triple frequency multipath analysis (Simsky, 2006) or furthermore triple frequency ambiguity resolution algorithms, like Three Carrier Ambiguity Resolution (TCAR) for Galileo or Cascading Integer Resolution (CIR) for GPS (Teunissen et al., 2002).

The goal of this work is to develop an improved TEC monitoring technique based on triple frequency GNSS measurements. In a first step, we have developed triple frequency simulation software, which enables us in a second step to develop and validate our TEC reconstruction technique on realistic GPS and Galileo measurements. The main advantage of this method is that, thanks to the availability of triple frequency measurements, TEC can be computed on the basis of two dual frequency GF phase combinations. Therefore, code measurements are only used in the preliminary step and do not affect the precision of the reconstructed TEC. As a consequence, the precision of the TEC is improved in regards with the usual dual frequency technique.

As the ionospheric delay—which mostly depends on TEC—remains the main limitation of GNSS precision and reliability, an improved monitoring of TEC will allow to increase the precision and reliability of several GNSS navigation and positioning techniques. Moreover, this improvement will open new opportunities for ionospheric studies.

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2. Triple frequency simulation software

As the third frequency is not yet available, we have developed software which allows to simulate realistic GNSS measurements. The final objective is to develop and to validate the TEC monitoring technique. As this TEC reconstruction technique uses both code and phase triple frequency (GPS and Galileo) measurements, the simulation software has to provide us with all those data types. Table 1 shows GPS and Galileo civil frequencies that will be used in this paper. For more readability, the E5b channel will be named L2, and the E5a channel L5, so that both GPS and Galileo frequencies will be named L1, L2, and L5.

2.1. Code and phase measurements

GNSS code measurements can be modeled as (Leick, 2004):

$$P^i_{p,k} = D^i_p + T^i_p + I^i_{p,k} - c\Delta t^i + c\Delta t_p + d^i_k + d_{p,k} + M^i_{p,k} + \sigma_{p,k} \quad (1)$$

This is expressed in meters, and i, p, k are the super- or subscripts, respectively, identifying the satellite, the receiver and the carrier frequency ($k = \text{L1}$, L2 or L5). Let us explain the different terms of Eq. (1):

- D^i_p: Geometric distance (vacuum distance) traveled by the signal between the satellite at transmission time and the receiver at reception time.
- T^i_p: Tropospheric delay. This delay is caused by the traveling of the signal through the neutral atmosphere. As it is a non-dispersive medium in regards with GNSS frequencies, this delay is independent of the carrier frequency.
- $I^i_{p,k}$: Ionospheric delay. This delay is caused by the traveling of the signal through the ionized atmosphere. As it is a dispersive medium in regards with GNSS frequencies, this delay is dependent on the carrier frequency. The ionospheric delay can be written as follows (Hofmann-Wellenhof et al., 1997):

  $$I^i_{p,k} = \frac{40.3}{f_k^2} 10^{16}\,\text{TEC} \quad (2)$$

  with TEC the total electron content, i.e. the integral of the electron concentration on the receiver-to-satellite path.

  In Eq. (2) as in the rest of this paper, TEC is given in TEC units or TECU, with $1\,\text{TECU} = 10^{16}\,\text{electrons/m}^2$.
- $c\Delta t^i$: Satellite clock error. This is the difference between the nominal time (i.e. the satellite clock reading) and the true time (i.e. in GNSS time scale) at transmission time. The term c is the velocity of light in vacuum, with $c = 299\,792\,458\,\text{m/s}$.
- $c\Delta t_p$: Receiver clock error. This is the difference between the nominal time (i.e. the receiver clock reading) and the true time (i.e. in GNSS time scale) at reception time.
- d^i_k: Satellite code hardware delay. This delay depends on the carrier frequency.
- $d_{p,k}$: Receiver code hardware delay. This delay also depends on the carrier frequency.
- $M^i_{p,k}$: Code multipath delay. This delay is periodic and is caused by signal reflections. Moreover, it depends on the satellite elevation.
- $\sigma_{p,k}$: Code measurement noise. This accidental error is linked to the measurement resolution and also depends on the satellite elevation.

Table 1
GPS and Galileo civil frequencies

GNSS system	Carrier signal	Frequency (MHz)
GPS	L1	1575.42
	L2	1227.60
	L5	1176.45
Galileo	L1	1575.42
	E5b	1207.14
	E5a	1176.45

Phase measurements can be modeled quite similarly (Leick, 2004):

$$\Phi^i_{p,k} = \frac{f_k}{c}(D^i_p + T^i_p - I^i_{p,k}) - f_k\Delta t^i + f_k\Delta t_p + \delta^i_k + \delta_{p,k} + m^i_{p,k} + s_{p,k} - N^i_{p,k} \quad (3)$$

This is expressed in cycles, which involves the introduction of the frequency f_k. This expression differs from Eq. (1) by the following terms:

- δ^i_k Satellite phase hardware delay
- $\delta_{p,k}$ Receiver phase hardware delay
- $m^i_{p,k}$ Phase multipath delay
- $s_{p,k}$ Phase measurement noise
- $N^i_{p,k}$ Integer ambiguity. It is the unknown integer constant representing the initial number of cycles (i.e. at the first epoch of observations) between the satellite and the receiver

Let us mention that phase hardware delays, phase multipath delays and phase measurement noise are lower than the corresponding code ones. As a consequence, although they are ambiguous, phase measurements are considered to be much more precise than code measurements.

2.2. Development of the software

In this section we will briefly explain how we have achieved the simulation of code and phase measurements as described in Eqs. (1) and (3). Let us explain *GPS*'s case. First, we have used several parameters transmitted in the real GPS navigation data in order to compute:

- the satellite position; it is computed in the Earth Centered Earth Fixed (ECEF) system by introducing the Keplerian parameters transmitted in the algorithm described in ICD-GPS-200C (2000); as a consequence, if the receiver coordinates are considered to be known, we can compute the term D_p^i;

- the ionospheric delay $I_{p,k}^i$ based on Klobuchar model; by using the eight Klobuchar coefficients we compute the Klobuchar TEC values which are introduced in Eq. (2); as the same ionospheric model is used in all steps of our TEC monitoring technique (developed in Section 3) whose unknown is the TEC, the use of this simple model will not have any influence on its results and its reliability; in fact, we will show that we are able to reconstruct TEC values whatever the ionospheric model used in the simulation software;
- the satellite clock error $c\Delta t^i$; it is computed by using the four clock coefficients and the relativistic effects parameters transmitted, as described in ICD-GPS-200C (2000);
- the satellite code hardware delay d_k^i; it is computed by using the Time Group Delay (or T_{GD}) transmitted.

Then we have computed T_p^i by applying a simple tropospheric correction model. In fact, we consider that zenith total delays (or ZTD) are constant (e.g. 2.40 m) and we use a simple mapping function (i.e. the cosine of the zenith angle z of the satellite) in order to obtain slant tropospheric delays (i.e. on the receiver-to-satellite path) by $T_p^i = ZTD/\cos z$. As all combinations used in Section 3 are independent of T_p^i, the use of this simple model will not have any influence on the reliability of the TEC monitoring technique.

The receiver clock error $c\Delta t_p$ is computed by using software based on real data. Although environmental conditions cause some slow drifts in the receiver code hardware delays, those drifts are slow enough to be neglected in this technique, so that we are allowed to simulate the delays $d_{p,k}$ as constant values. The code multipath delay $M_{p,k}^i$ is computed on the basis of a Gaussian time-correlated noise modulated by several characteristics: the amplitude, the environment factor, the elevation factor, etc. Quite similarly, the code measurement noise $\sigma_{p,k}$ is simulated as a Gaussian noise characterized by its zero-mean and its standard deviation.

As far as the phase observables are concerned, we introduce the integer ambiguity $N_{p,k}^i$ by computing an integer constant value at each period for each satellite. Besides, we do not simulate satellite and receiver phase hardware delays (δ_k^i and $\delta_{p,k}$), phase multipath delays ($m_{p,k}^i$) and phase measurement noise ($s_{p,k}$). It will be first assumed that all those delays are negligible with respect to our TEC monitoring technique developed below, so that we do not introduce them in Eq. (3). But then at each step we will study their influence to see whether we are allowed to neglect them or not.

Let us now come to *Galileo*'s case. We will only mention what it is different from GPS's case. Firstly, as there is only one Galileo test satellite in orbit at the moment (GIOVE-A), we are not able to use real transmitted navigation data in our simulation software. However, we already know how the constellation is going to be built. By using the parameters described in SIS-ICD (2007) in software based on Keplerian laws, we have simulated 30 satellites equally distributed and spaced on three orbital planes. Their inclination equals 56° and the semi-major axis 29 600 km.

Moreover, we consider that vertical TEC values TEC_V are constant, so that the ionospheric delay $I_{p,k}^i$ is computed as follows:

$$I_{p,k}^i = \frac{40.3}{f_k^2} 10^{16} \frac{TEC_V}{\cos z_{IP}} \tag{4}$$

with $\cos z_{IP}$ the mapping function used to convert the vertical values (TEC_V) into slant values (TEC—Eq. (2)), and z_{IP} the zenith angle of the satellite at the ionospheric point. Let us note that to obtain z_{IP} we use a single layer ionospheric model with the ionospheric shell height fixed at 350 km (Hofmann-Wellenhof et al., 1997). Like in GPS's case, as the same ionospheric model is used in all steps of our method (developed in Section 3) whose unknown is the TEC, the use of this simple model will not have any influence on its results and its reliability.

In addition, as there are no real Galileo data available, three different parameters cannot be simulated like in GPS's case: the satellite clock error, the receiver clock error and the satellite code hardware delay. As all combinations used in Section 3 to monitor the TEC are independent of the first two parameters ($c\Delta t^i$ and $c\Delta t_p$), we are allowed not to include them in Eqs. (1) and (3). Moreover, with the same assumptions than in the receiver's case (see above), we are allowed to simulate the third parameter d_k^i as a constant value.

Our simulation software has been validated by comparing simulated and real dual frequency GPS data. As they are developed on the same basis, it will be assumed that the triple frequency GPS and Galileo software are also valid.

3. Triple frequency TEC monitoring technique

Our final objective is to develop a triple frequency TEC monitoring technique. It is important to note that the monitoring technique requires the use of code and phase measurements coming from one station only, and not double differenced measurements like in RTK positioning techniques for example. First we outline "theoretical" aspects of our three-step TEC reconstruction technique (see Sections 3.1–3.3), and then we validate it based on the triple frequency simulation software (see Section 3.4).

3.1. First step

The objective of the first step is to resolve the extra-widelane (EWL) ambiguities N_{25} (in cycles):

$$N_{25} = N_{p,L5}^i - N_{p,L2}^i \tag{5}$$

These ambiguities are integer numbers and can be estimated by computing the extra-widelane-narrowlane (EWLNL) combination C_{25}, i.e. by making the difference between the EWL combination of L2 and L5 phase measurements and the narrowlane combination of L2 and L5 code measurements (in cycles):

$$C_{25} = \Phi^i_{p,\mathrm{L2}} - \Phi^i_{p,\mathrm{L5}} - \frac{f_{\mathrm{L2}} - f_{\mathrm{L5}}}{f_{\mathrm{L2}} + f_{\mathrm{L5}}} \times \left(\frac{f_{\mathrm{L2}}}{c} P^i_{p,\mathrm{L2}} + \frac{f_{\mathrm{L5}}}{c} P^i_{p,\mathrm{L5}}\right) \quad (6)$$

The wavelength of this combination is given by $\lambda_{25} = c/(f_{\mathrm{L2}} - f_{\mathrm{L5}})$ and equals 5.861 m for GPS and 9.768 m for Galileo.

If we introduce Eqs. (1) and (3) in Eq. (6), we obtain the EWL ambiguities N_{25} plus a residual term ΔR_{25}:

$$C_{25} = N_{25} + \Delta R_{25} \quad (7)$$

This residual term depends on satellite and receiver hardware delays, multipath delays and measurement noise on both code and phase measurements for L2 and L5 frequencies, and can be written as follows (in cycles):

$$\begin{aligned}\Delta R_{25} &= \Delta R'_{25} + \Delta R''_{25} + \Delta R'''_{25} + \Delta R''''_{25} \\ &= -f_{25}\left[\frac{f_{\mathrm{L2}}}{c}(M^i_{p,\mathrm{L2}} + \sigma^i_{p,\mathrm{L2}}) + \frac{f_{\mathrm{L5}}}{c}(M^i_{p,\mathrm{L5}} + \sigma^i_{p,\mathrm{L5}})\right] \\ &\quad - f_{25}\left[\frac{f_{\mathrm{L2}}}{c}(m^i_{p,\mathrm{L2}} + s^i_{p,\mathrm{L2}}) + \frac{f_{\mathrm{L5}}}{c}(m^i_{p,\mathrm{L5}} + s^i_{p,\mathrm{L5}})\right] \\ &\quad - f_{25}\left[\frac{f_{\mathrm{L2}}}{c}(d^i_{\mathrm{L2}} + d_{p,\mathrm{L2}}) + \frac{f_{\mathrm{L5}}}{c}(d^i_{\mathrm{L5}} + d_{p,\mathrm{L5}})\right] \\ &\quad - f_{25}\left[\frac{f_{\mathrm{L2}}}{c}(\delta^i_{\mathrm{L2}} + \delta_{p,\mathrm{L2}}) + \frac{f_{\mathrm{L5}}}{c}(\delta^i_{\mathrm{L5}} + \delta_{p,\mathrm{L5}})\right] \end{aligned} \quad (8)$$

with $f_{25} = (f_{\mathrm{L2}} - f_{\mathrm{L5}})/(f_{\mathrm{L2}} + f_{\mathrm{L5}})$.

We have to determine whether it is possible to resolve the EWL ambiguities—i.e. to fix them at their correct integer values—despite the existence of the residual term ΔR_{25}. For that purpose, we have to estimate whether the influence of that residual term could exceed half a cycle. We can make the assumption that satellite and receiver code and phase hardware delays are constant in time (see Section 2.2), so that we can consider $\Delta R'''_{25}$ and $\Delta R''''_{25}$ as constant terms. Therefore we have first estimated the influence of the variable part of Eq. (8) on the resolution of the EWL ambiguities N_{25}, i.e. the influence of *multipath delays* and *measurement noise*. We neglect the influence of phase multipath delays and phase measurement noise (noted $\Delta R''_{25}$) with respect to the influence of the corresponding *code* delays (Ciraolo et al., 2007), so that we will only consider this last influence (noted $\Delta R'_{25}$).

Fig. 1, respectively, shows representative GPS's (a) and Galileo's (b) cases of $\Delta R'_{25}$ in function of time. Because of the properties of code multipath and code noise (see Section 2.2), this is very variable. Moreover, we can see that this term does not exceed half a cycle in all GPS's and Galileo's cases studied, but is by far smaller for Galileo ($\leqslant$0.1 cycle) than for GPS ($\leqslant$0.25 cycle).

Let us see how it is possible to explain those results, and particularly such a difference between GPS and Galileo performance.

Firstly, the most important part of the difference can be explained by the fact that the EWLNL combination wavelength λ_{25} is larger for Galileo (GA) than for GPS

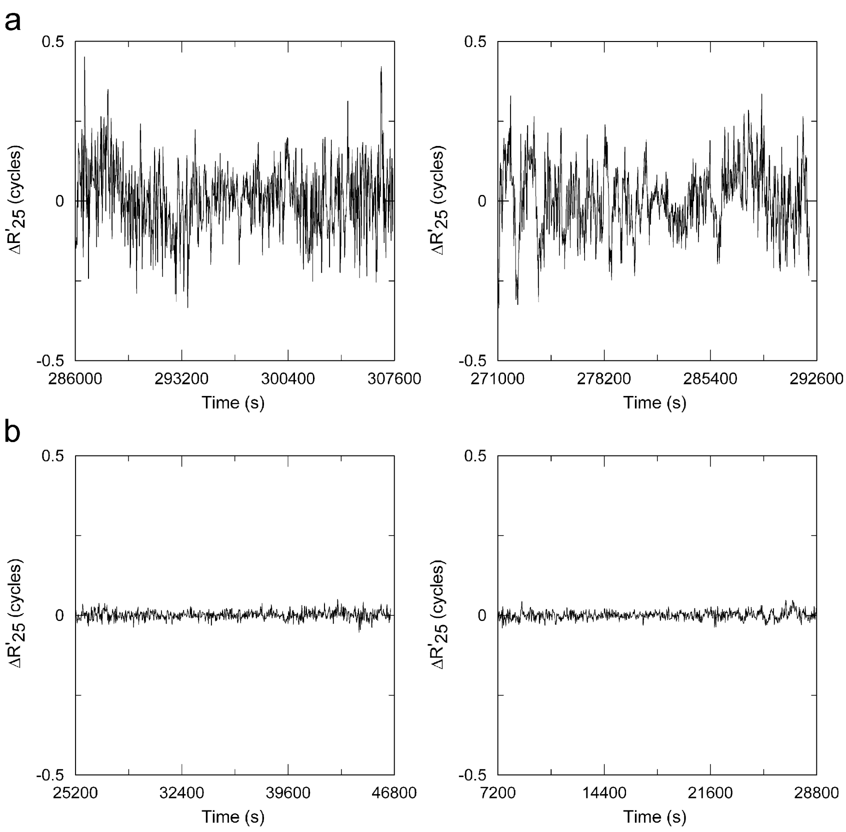

Fig. 1. Influence of code multipath delays and code measurement noise on the EWL ambiguities N_{25} (ΔR_{25}) for two GPS (a) and Galileo (b) satellites.

(GP), as $\lambda_{25-GA}/\lambda_{25-GP} = 1.66$. This means that if the considered residual terms ($M^i_{p,L2}$, $M^i_{p,L5}$, $\sigma^i_{p,L2}$ and $\sigma^i_{p,L5}$) had exactly the same importance in both cases, $\Delta R'_{25}$ would be 1.66 times smaller for Galileo than for GPS. To prove it, we can compute the following ratio from Eq. (8):

$$f_{25}\left(\frac{f_{L2}}{c}+\frac{f_{L5}}{c}\right)_{GP}\Big/ f_{25}\left(\frac{f_{L2}}{c}+\frac{f_{L5}}{c}\right)_{GA} = 1.66$$

Secondly, the difference can also be explained by the fact that the amplitude of code multipath and the standard deviation of code noise are lower for Galileo than for GPS. Table 2 presents the corresponding simulated values, which were chosen on the basis of observed values (for GPS L1 and L2 frequencies) and of realistic predicted values (for GPS L5 and all Galileo frequencies). As explained in Simsky et al. (2005), those values are lower for Galileo thanks to a new code modulation scheme and thanks to the fact that the power of Galileo signals will be twice as great as the power of GPS signals.

In summary, without taking into account the influence of code hardware delays, we can affirm that it is easier to resolve Galileo EWL ambiguities than GPS ones.

Secondly let us take into account the influence of *satellite* and *receiver code and phase hardware delays*, i.e. of the sum of $\Delta R'''_{25}$ and $\Delta R''''_{25}$. In order to resolve the EWL ambiguities N_{25}, $(\Delta R'''_{25} + \Delta R''''_{25})$ should be lower than the margin let by $\Delta R'_{25}$ in regards with half a cycle. In this context, we would need to have a precise idea about the amplitude of GPS and Galileo $\Delta R'''_{25}$ and $\Delta R''''_{25}$ terms. However, at the moment, we are only able to estimate the influence of GPS satellite code hardware delays (d^i_{L1} and d^i_{L2}) on the basis of T_{GD} transmitted values, as seen in Section 2.2. But we do not have many indications about the amplitude of GPS receiver code hardware delays ($d_{p,k}$), as well as about the future Galileo—satellite and receiver—code hardware delays. Furthermore, it is difficult to guess the amplitude of GPS and Galileo satellite and receiver phase hardware delays. As a consequence, we would at least need real triple frequency GNSS data to begin a more detailed analysis about the influence of hardware delays.

Nevertheless, we can try to discuss the influence of $(\Delta R'''_{25} + \Delta R''''_{25})$ in another way. The previous results allow us to consider that in most of the cases the influence of code multipath delays and code measurement noise (named $\Delta R'_{25,m}$) is lower than 0.25 cycle for GPS and 0.1 cycle for Galileo. As a consequence, the remaining margin let for the code and phase hardware delays is named $(\Delta R'''_{25} + \Delta R''''_{25})_m$ and equals $0.5 - \Delta R'_{25,m}$, i.e., respectively, 0.25 and 0.4 cycle. By making some assumptions, we can then compute how large code and phase hardware delays can be without exceeding this margin. Let us first consider that the influence of $(\Delta R'''_{25} + \Delta R''''_{25})$ is equally distributed to both frequencies L2 and L5. By using Eq. (8), we find that $(d^i_{L2} + d_{p,L2} + \delta^i_{L2} + \delta_{p,L2})$ and $(d^i_{L5} + d_{p,L5} + \delta^i_{L5} + \delta_{p,L5})$ have to be both smaller than ± 1.50 m (or ± 5 ns) for GPS and smaller than ± 4 m (or ± 13 ns) for Galileo. Secondly, if we consider that the influence of $(\Delta R'''_{25} + \Delta R''''_{25})$ is equally distributed to satellite hardware delays on one side and receiver hardware delays on the other side, we find very similar conditions for the maximal amplitude of code and phase hardware delays.

Further, if we consider that navigation data will allow us to introduce approximated values of d^i_{L2} and d^i_{L5} in Eq. (8), and therefore to reduce the influence of $\Delta R'''_{25}$, we can reasonably affirm that the conditions defined here above can be satisfied—particularly for Galileo—so that the influence of code and phase hardware delays $(\Delta R'''_{25} + \Delta R''''_{25})$ does not exceed $(\Delta R'''_{25} + \Delta R''''_{25})_m$.

As a consequence, if we put the *two steps* of the analysis together, we can consider that ΔR_{25} does not exceed half a cycle, so that it is possible to resolve the EWL ambiguities N_{25}. Even if this assumption seems reasonable from the previous discussion, it will be necessary to verify its validity when real triple frequency data will be used to reconstruct TEC.

3.2. *Second step*

The objective of the second step is to resolve the widelane (WL) ambiguities N_{12} (in cycles):

$$N_{12} = N^i_{p,L2} - N^i_{p,L1} \tag{9}$$

As EWL ambiguities, these ambiguities are also integer numbers. They can be estimated similarly by computing the widelane-narrowlane (WLNL) combination C_{12}, which is the difference between the WL combination of L1 and L2 phase measurements and the narrowlane combination of L1 and L2 code measurements. The wavelength of the WLNL combination is given by $\lambda_{12} = c/(f_{L1} - f_{L2})$ and equals 0.862 m for GPS and 0.814 m for Galileo. In a similar way to Eqs. (6) and (7), we obtain:

$$C_{12} = \Phi^i_{p,L1} - \Phi^i_{p,L2} - \frac{f_{L1} - f_{L2}}{f_{L1} + f_{L2}} \times \left(\frac{f_{L1}}{c}P^i_{p,L1} + \frac{f_{L2}}{c}P^i_{p,L2}\right) = N_{12} + \Delta R_{12} \tag{10}$$

Moreover, the residual term ΔR_{12} can be built as ΔR_{25} (see Eq. (8)), except that satellite and receiver hardware delays, multipath delays and measurement noise are related to L1 and L2 frequencies:

$$\Delta R_{12} = \Delta R'_{12} + \Delta R''_{12} + \Delta R'''_{12} + \Delta R''''_{12} \tag{11}$$

Let us compare the WLNL combination performance with the EWLNL combination one. Firstly the WLNL combination wavelength (λ_{12}) is by far smaller than the EWLNL combination wavelength (λ_{25}). In fact, we obtain $\lambda_{25}/\lambda_{12} =$

Table 2
Code multipath delays ($M^i_{p,k}$) amplitude and code measurement noise ($\sigma^i_{p,k}$) standard deviation for GPS and Galileo frequencies

Carrier signal	$M^i_{p,k}$ amplitude (m)	$\sigma^i_{p,k}$ standard deviation (m)
L1	2.00	0.30
L2	2.00	0.30
L5	0.50	0.10
L1	0.20	0.17
E5b	0.20	0.11
E5a	0.20	0.11

6.8 for GPS and $\lambda_{25}/\lambda_{12} = 12$ for Galileo. Therefore, if the different delays included in Eq. (11) had the same amplitude than those included in Eq. (8), ΔR_{12} would be 6.8 and 12 times larger than ΔR_{25}, respectively, for GPS and Galileo. Furthermore, in regards to Table 2, the amplitude of code multipath as well as the standard deviation of code noise are larger for the WLNL's case than for the EWLNL's case—especially for GPS—which makes ΔR_{12} (through $\Delta R'_{12}$) once more larger than ΔR_{25}. Fig. 2 shows $\Delta R'_{12}$—i.e. the influence of code multipath delays and code measurement noise on N_{12}—in function of time for GPS (a) and Galileo (b) for the same cases than in Fig. 1. We can see that $\Delta R'_{12}$ already exceeds 2 cycles for GPS and that it can reach 0.5 cycle for Galileo. Therefore, when taking into account the additional influence of $\Delta R'''_{12}$ (code hardware delays) but also of $\Delta R''_{12}$ and $\Delta R''''_{12}$ (phase delays), ΔR_{12} will automatically exceed half a cycle. As a consequence, WL ambiguities N_{12} cannot be resolved by using the WLNL combination.

For this reason, we have to form another combination called differenced widelane (DWL) combination C_{125} and built as follows (in cycles):

$$C_{125} = (\Phi^i_{p,\text{L1}} - \Phi^i_{p,\text{L2}}) - (\Phi^i_{p,\text{L2}} - \Phi^i_{p,\text{L5}} + N_{25})\frac{\lambda_{25}}{\lambda_{12}} \tag{12}$$

The DWL combination includes the difference of L1 and L2 phase measurements $\Phi^i_{p,\text{L1}} - \Phi^i_{p,\text{L2}}$, as well as the difference of L2 and L5 phase measurements $\Phi^i_{p,\text{L2}} - \Phi^i_{p,\text{L5}}$, respectively, called WL and EWL combinations. Let us mention that WL and EWL combination wavelengths correspond to the WLNL and EWLNL combination wavelengths λ_{12} and λ_{25} defined in Sections 3.1 and 3.2, so that they can be used in Eq. (12). Moreover, the DWL combination is based on the EWL ambiguities N_{25} which are considered as resolved from the first step (see Section 3.1). From Eqs. (3) and (5) it comes that from Eq. (12) we obtain the WL ambiguities N_{12} plus a residual term ΔR_{125}:

$$C_{125} = N_{12} + \Delta R_{125} \tag{13}$$

This residual term can be written as follows (in cycles):

$$\Delta R_{125} = I^i_{p,\text{L1}} \times b \tag{14}$$

with $I^i_{p,\text{L1}}$ the ionospheric delay on L1 and b a constant term which depends on the three frequencies as follows:

$$b = \frac{f_{\text{L1}}f_{\text{L2}}f_{\text{L5}} - f^2_{\text{L1}}f_{\text{L2}} + f^3_{\text{L1}} - f^2_{\text{L1}}f_{\text{L5}}}{cf_{\text{L2}}f_{\text{L5}}}$$

The b term approximately equals $0.5\,\text{m}^{-1}$. As with a low solar activity TEC mid-latitude value ($\text{TEC}_\text{V} = 20$ TECU) given by Warnant and Pottiaux (2000), the value of $I^i_{p,\text{L1}}$ (with $\cos z_{\text{IP}} = 1$) computed by Eq. (4) equals 3.25 m, the residual term ΔR_{125} will be greater than 1.6 cycles. With higher solar activity or lower latitude TEC_V values (i.e. higher TEC_V values), ΔR_{125} will be even greater. As a

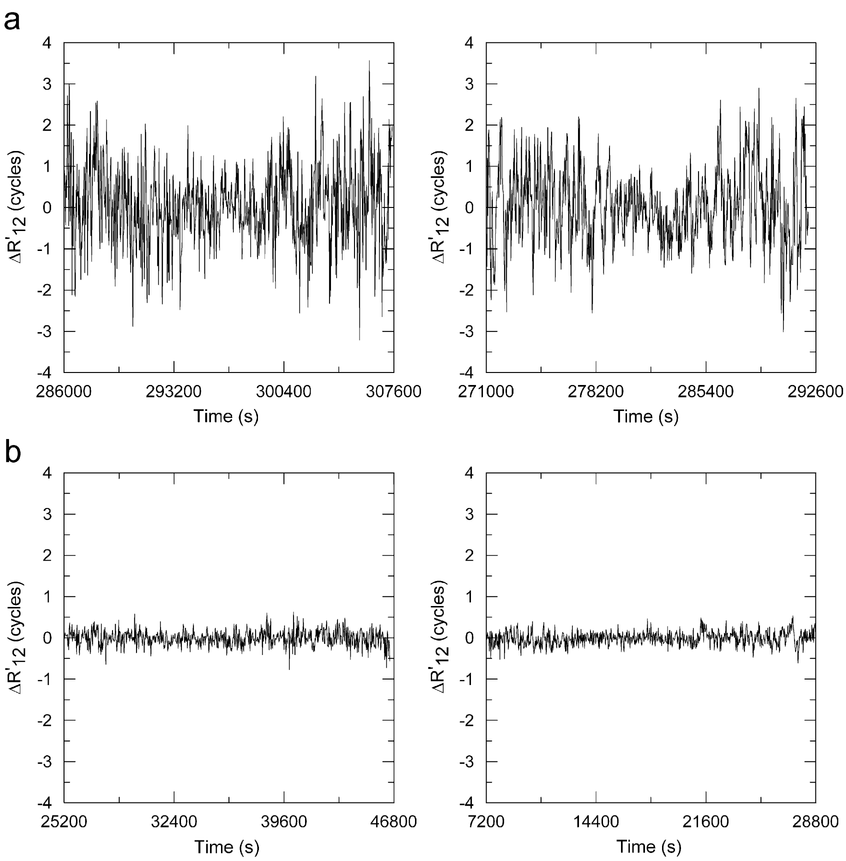

Fig. 2. Influence of code multipath delays and code measurement noise on the WL ambiguities N_{12} (ΔR_{12}) for two GPS (a) and Galileo (b) satellites.

consequence, ΔR_{125} will in any case cause an error of several cycles on N_{12}.

In other words, even with the use of the DWL combination C_{125}, it is impossible to fix the WL ambiguities N_{12} at their correct integer values. Nevertheless, DWL combination gives an "approximated" integer value of N_{12}, i.e. an integer value which corresponds to the correct integer value plus several cycles corresponding to ΔR_{125}. This approximated integer value of N_{12} is used in the third step of the TEC monitoring technique (see Section 3.3).

Let us mention that Eq. (14) should normally include a term combining phase hardware delays, phase multipath delays and phase measurement noise. However, even if phase delays would involve a modification of the "approximated" integer value of N_{12} (therefore by an integer number of cycles), it would not have any influence on the final resolution of the WL ambiguities which is processed in the third step. Let us explain why: a change of an integer number of cycles (e.g. $+n$ cycles) in the "approximated" integer value of N_{12} causes a change of $-n * n_{11}$ TECU in the TEC_a values (computed by Eq. (17)). Therefore the term $\Delta N_{12-\text{e}}$ which is used to find the correct value of N_{12} changes by $-n$ cycles, which compensates the initial change of $+n$ cycles.

Finally let us already note that an increase of the ionospheric ionization level will not have any influence on the results of our TEC monitoring technique, unless it is linked with a critical dual frequency TEC precision. This statement will be detailed in Section 3.4.

3.3. Third step

The objective of the third step is to use the results of the first two steps in order to achieve the monitoring of the TEC. The availability of triple frequency measurements allows to form two independent dual frequency combinations, so that TEC can be reconstructed directly based on two GF phase combinations $\Phi^i_{p,\text{GF}}$ and $\Phi^i_{p,\text{GF}'}$. Let us mention that $\Phi^i_{p,\text{GF}}$ is the GF phase combination used in the usual dual frequency TEC monitoring technique (see Section 1).

$$\begin{cases} \Phi^i_{p,\text{GF}} = \Phi^i_{p,\text{L1}} - c_{12}\Phi^i_{p,\text{L2}} \\ \Phi^i_{p,\text{GF}'} = \Phi^i_{p,\text{L2}} - c_{25}\Phi^i_{p,\text{L5}} \end{cases} \tag{15}$$

with $c_{12} = f_{\text{L1}}/f_{\text{L2}}$ and $c_{25} = f_{\text{L2}}/f_{\text{L5}}$.

If we introduce $\Phi^i_{p,\text{L1}}$, $\Phi^i_{p,\text{L2}}$ and $\Phi^i_{p,\text{L5}}$ in Eq. (15) by using Eq. (3)—with neglecting all phase delays—we obtain

$$\begin{cases} \Phi^i_{p,\text{GF}} = a_{12}\,\text{TEC} - N^i_{p,\text{L1}} + c_{12}N^i_{p,\text{L2}} \\ \Phi^i_{p,\text{GF}'} = a_{25}\,\text{TEC} - N^i_{p,\text{L2}} + c_{25}N^i_{p,\text{L5}} \end{cases} \tag{16}$$

with

$$a_{12} = 40.3 \times 10^{16}(f_{\text{L1}}/c)(1/f^2_{\text{L2}} - 1/f^2_{\text{L1}}),$$
$$a_{25} = 40.3 \times 10^{16}(f_{\text{L2}}/c)(1/f^2_{\text{L5}} - 1/f^2_{\text{L2}})$$

Eq. (16) shows that all the terms relative to the geometry, clocks, etc. disappear, so that the TEC and the three ambiguities are the only remaining unknowns. As there are four unknowns, the system cannot be resolved without additional information. However, the values of N_{25} (resolved in the first step—see Section 3.1) and the values of N_{12} (approximated in the second step—see Section 3.2) can be introduced in Eq. (16), so that there remain only two unknowns. In practice, by using Eqs. (5) and (9) in Eq. (16) we can substitute $N^i_{p,\text{L1}}$ for $(N^i_{p,\text{L2}} - N_{12})$ and $N^i_{p,\text{L5}}$ for $(N^i_{p,\text{L2}} + N_{25})$. By doing so, we can express the unknowns TEC and $N^i_{p,\text{L2}}$ in function of the observations in a matrix form:

$$\begin{pmatrix} \text{TEC} \\ N^i_{p,\text{L2}} \end{pmatrix} = \begin{pmatrix} a_{12} & c_{12}-1 \\ a_{25} & c_{25}-1 \end{pmatrix}^{-1} \begin{pmatrix} \Phi^i_{p,\text{GF}} - N_{12} \\ \Phi^i_{p,\text{GF}'} - c_{25}N_{25} \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \begin{pmatrix} \Phi^i_{p,\text{GF}} - N_{12} \\ \Phi^i_{p,\text{GF}'} - c_{25}N_{25} \end{pmatrix} \tag{17}$$

We are now able to resolve the system. But as the WL ambiguities N_{12} coming from Section 3.2 are "approximated" integer values, the TEC and $N^i_{p,\text{L2}}$ resulting values are also "approximated".

Nevertheless, on the basis of two properties, we are able to fix N_{12} at their correct integer values, so that we obtain the correct values of the two unknowns. The first property is that WL ambiguities are integer numbers, which makes their resolution easier. The second property is that in Eq. (17) a change of 1 cycle in the WL ambiguities causes a change of approximately 12 TECU in the TEC values. In fact, the term n_{11}—by which is multiplied N_{12} to compute the TEC—equals -12.196 TECU/cycle for GPS and -11.327 TECU/cycle for Galileo. Those two properties can be used as follows in order to resolve N_{12}, and therefore to obtain the correct TEC and ambiguities values.

(1) We introduce the "approximated" values of N_{12} obtained from Eq. (13) in Eq. (17) in order to compute the "approximated" values of TEC, named TEC_a. The correct TEC values will be named TEC_b.
(2) We use the usual dual frequency TEC monitoring technique in order to compute a rough estimation of TEC values (i.e. of TEC_b) named TEC_e. This technique allows to reconstruct TEC with a precision of 2–3 TECU (see Section 1). Thanks to lower code multipath delays, this precision could even be better in Galileo's case.
(3) We compute the difference between the "approximated" and estimated values of TEC: $\Delta\text{TEC}_\text{e} = \text{TEC}_\text{a} - \text{TEC}_\text{e}$. That gives us an approximation of the real error made on TEC values, i.e. of $\Delta\text{TEC}_\text{b} = \text{TEC}_\text{a} - \text{TEC}_\text{b}$.
(4) We use the two properties explained here above. Using the second property, we compute $\Delta N_{12-\text{e}} = \Delta\text{TEC}_\text{e}/n_{11}$ which is an approximation of the real error made on the WL ambiguities ($\Delta N_{12-\text{b}} = \Delta\text{TEC}_\text{b}/n_{11}$). Let us prove that this approximation is precise enough to resolve the WL ambiguities N_{12}. In other words, as N_{12} are integer values (first property), we have to prove that the difference between $\Delta N_{12-\text{e}}$ and $\Delta N_{12-\text{b}}$

is smaller than 0.5 cycle. This can be written as

$$\Delta N_{12-e} - \Delta N_{12-b} = \frac{\Delta TEC_e}{n_{11}} - \frac{\Delta TEC_b}{n_{11}} < \frac{1}{2}$$

From the previous considerations we have $\Delta TEC_e - \Delta TEC_b = TEC_b - TEC_e$, so that the condition above becomes $TEC_b - TEC_e < \frac{1}{2} n_{11}$ which approximately equals 5–6 TECU. Regarding to the usual mid-latitude precision of TEC_e (2–3 TECU or even better for Galileo), this condition is fullfilled. However, the typical error might be higher in daytime equatorial conditions. In that case, if the error on the estimated TEC is larger than 5–6 TECU, it would not be possible to find the correct values of the WL ambiguities and therefore to compute the correct TEC values.

(5) We introduce the correct values of N_{12} in Eq. (17), so that we finally obtain the correct values of TEC (TEC_b) and the correct values of $N^i_{p,L2}$. If we use the final values of N_{25} (from Section 3.1), N_{12} and $N^i_{p,L2}$ (both from this section) in Eqs. (5) and (9), we also obtain the correct values of $N^i_{p,L1}$ and $N^i_{p,L5}$. In other words, we are able to reconstruct the correct TEC values and to resolve the three phase ambiguities.

As we are able to reconstruct TEC values exclusively on the basis of two GF phase combinations by Eq. (15), their precision is not affected by code hardware delays neither by code multipath delays and corresponds to the precision of phase measurements. However, let us mention that we have neglected phase delays in Eq. (3) and therefore in Eq. (15). This assumption could have an influence on the resolution of the GF system of Eq. (17). Let us consider that Δd_k is the sum of all phase delays—hardware, multipath and measurement noise—on each carrier frequency (in cycles):

$$\Delta d_k = \delta^i_k + \delta_{p,k} + m^i_{p,k} + s_{p,k}$$

As a consequence, the error caused by Δd_{L1}, Δd_{L2} and Δd_{L5} on TEC values can be written as follows (in TECU):

$$\begin{aligned} e_{TEC} &= n_{11}(\Delta d_{L1} - c_{12}\Delta d_{L2}) + n_{12}(\Delta d_{L2} - c_{25}\Delta d_{L5}) \\ &= n_{11}\Delta d_{L1} + (n_{12} - n_{11}c_{12})\Delta d_{L2} - n_{12}c_{25}\Delta d_{L5} \end{aligned} \quad (18)$$

However, even if we can assume that phase multipath delays and phase measurement noise have a millimeter-level amplitude (Ciraolo et al., 2007), it remains very difficult to estimate the total amplitude but also the resulting sign of Δd_{L1}, Δd_{L2} and Δd_{L5}. Therefore, e_{TEC} cannot be clearly estimated. Nevertheless, we will guess the importance of e_{TEC} on the basis of definite assumptions. For more intelligibility, let us give the values of the coefficients of Eq. (18), resp. for GPS and Galileo: n_{11} equals $-12.196/-11.327$, $(n_{12} - n_{11}c_{12})$ equals 95.135/147.254 and $n_{12}c_{25}$ equals 82.939/135.927 TECU/cycle. Taking those values into account, we can affirm that:

- if $\Delta d_{L1} = \Delta d_{L2} = \Delta d_{L5}$ (whatever their amplitude), e_{TEC} equals zero TECU;
- if the sign of Δd_{L2} is the same as Δd_{L5}'s one, it minimizes e_{TEC};
- on the contrary, if the sign of Δd_{L2} is opposite to Δd_{L5}'s one, it maximizes e_{TEC}, especially because of the lower amplitude of Δd_{L1}.

By doing some tests with different millimeter-level amplitude and different sign values of Δd_{L1}, Δd_{L2} and Δd_{L5}, we observe that e_{TEC} usually equals several tenth of TECU. In conclusion, even if the precision of the triple frequency reconstructed TEC is affected by phase delays, we can expect that it will be improved in regards with the usual dual frequency technique. However, this has to be quantified and confirmed by testing the technique on real data.

3.4. Validation

Next to the development of "theoretical" aspects of the TEC monitoring technique, we have to validate it on triple frequency GPS and Galileo simulated data. This validation approach can be divided in two parts: first we apply the three steps of the method on code and phase simulated data, and then we verify the results of each step with the appropriated simulated quantities. Code and phase measurements used in this validation step are simulated as described in Section 2.2.

For the validation of the first step, we proceed the EWLNL combination (Eq. (6)) in order to see whether the EWL ambiguities N_{25} can be resolved at their correct integer values despite the existence of the residual term ΔR_{25}. If we compare the results obtained with the initial simulated ambiguities, we show that it is indeed possible within several epochs of observations. In addition, as already seen through the analysis of Figs. 1 and 2, the resolution is more efficient for Galileo than for GPS. This can be explained by the larger EWL combination wavelength and by the lower values of code multipath delays and code measurement noise. Nevertheless, let us remember that those results are exclusively based on simulated measurements and should be validated on real data, especially as far as code hardware delays are concerned.

For the validation of the second step, we proceed the DWL combination (Eq. (12)). As this step is clearly linked with the third one, it cannot really be validated individually. However, the results already show that as expected the "approximated" integer values of N_{12} obtained equal the correct integer values plus 2 or 3 cycles.

Finally, for the validation of the third step, we proceed the GF phase combinations (Eq. (17)) by introducing the resolved values of N_{25} and the "approximated" values of N_{12}. By doing so, the approach described in Section 3.3 shows good results. In fact, we finally obtain the correct TEC values (TEC_b), i.e. the values initially simulated for GPS by the Klobuchar model or for Galileo by the constant model (see Section 2.2). As Eq. (17) does not take into account phase delays, we obtain exactly the same TEC values as initially simulated.

Let us mention that for the initial validation of Galileo's TEC monitoring technique, we use a low ionization level (i.e. $TEC_V = 20$ TECU). Further we have tested higher TEC_V

values (up to 200 TECU) to see whether the basic ionization level of the ionosphere has an influence on the results of our TEC monitoring technique. This is tested on Galileo but the results will also be applicable to GPS. The computed results show that whatever the ionization level applied we obtain the TEC values initially simulated, which means that this has no influence on the results. Let us explain those observed results in detail in regards with the three steps. Considering Eq. (7) it is clear that it has no influence on the first step. As far as the second step is concerned, an increase of TEC_V values involves an increase of the residual term ΔR_{125}. In other words, the "approximated" integer values of N_{12} obtained for Eq. (13) as well as "approximated" TEC and $N^i_{p,L2}$ values obtained from Eq. (17) increasingly move away from their respective correct values. Nevertheless, we have already seen that a modification of the "approximated" integer value of N_{12} does not have any influence on the final resolution of the WL ambiguities processed in the third step (see Section 3.2). As far as the third step is concerned, the method applied to obtain the correct TEC and ambiguities values is based on the use of estimated TEC values (TEC_e). It is known that their precision is essentially limited by code multipath delays and differential satellite and receiver code hardware delays but does not depend directly on the ionization level of the ionosphere. As a consequence, unless it would involve an error on TEC_e higher than 5–6 TECU (see Section 3.3), we can conclude that the ionization level of the ionosphere does not have any influence on the results.

4. Conclusions

This paper has introduced an improved TEC monitoring technique based on triple frequency GNSS—GPS and Galileo—measurements. This technique is divided in three steps. In the first step, the EWLNL combination enables to fix the EWL ambiguities N_{25}. In the second step, the DWL combination gives an "approximated" integer value of the WL ambiguities N_{12}. In the third step, thanks to the availability of triple frequency data, we form two independent dual frequency GF phase combinations. By introducing the values of N_{25} (resolved) and of N_{12} (approximated) in those combinations, we only find approximated values of the two remaining unknowns TEC and $N^i_{p,L2}$. Nevertheless, on the basis of two properties and using a rough estimation of TEC, we are able to fix the WL ambiguities at their correct integer values. As a consequence, we obtain the correct integer values for the ambiguities and we are able to reconstruct TEC.

The triple frequency software that we have developed enables us to validate the TEC monitoring technique on realistic GPS and Galileo measurements. We show that we are able to precisely reconstruct the simulated TEC values. However, the technique is more reliable for Galileo than for GPS. This can be explained by the larger EWL combination wavelength and by the properties of code multipath delays and code measurement noise. Besides we have shown that the ionization level of the ionosphere does not have any influence on the results.

As code measurements are only used in the first step of the technique, the precision of the reconstructed TEC is not affected by code hardware delays neither by code multipath delays. Furthermore, even if the reconstructed TEC is affected by phase delays, we can expect that the precision will be improved in regards with the usual dual frequency technique.

Let us finally give some perspectives for our work. The availability of real Galileo data from Giove-A satellite, and particularly triple frequency (L1-E5a-E5b) code and phase measurements, will allow to validate our TEC monitoring technique on the three-step level and to validate our assumptions.

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