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GEOPHYSIQUE
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A NEW COORDINATE CHANGE IN MATHEMATICAL MODELLING OF THE UPPER IONOSPHERE

S. M. Stankov

(Submitted by Corresponding Member G. Nestorov on May 30, 1990)

The mathematical modelling of the ionospheric *F*-region imposes the solution of differential equations along the plasma tubes of the type

(1) $$\frac{dG}{dt}=A_1\frac{\partial^2 G}{\partial s^2}+A_2\frac{\partial G}{\partial s}+A_3G+A_4.$$

The coordinate system (L, s, t) associated with (1) is not orthogonal and this causes some difficulties of computational nature. The coordinates (L, s, t) can be substituted for the curvilinear coordinates (p, q, t) proposed by Kendall [3] which are defined by the spherical polar coordinates (r, θ, t) as follows:

(2) $$p=\frac{r}{r_0\sin^2\theta},\quad q=\frac{r_0^2\cos\theta}{r^2}.$$

where r is the radial distance to the Earth's centre, r_0 is the Earth's radius, θ is the colatitude and t is the time.

Thus, equation (1) acquires the form

(3) $$\frac{dG}{dt}=B_1\frac{\partial^2 G}{\partial q^2}+B_2\frac{\partial G}{\partial q}+B_3G+B_4,$$

where

$$B_1=A_1\eta^2$$
$$B_2=A_1\eta\frac{\partial\eta}{\partial q}+A_2\eta$$
$$B_i=A_i,\quad i=3,\ 4$$
$$\eta=-(1+3\cos^2\theta)^{1/2}r_0^2/r^3.$$

The coordinate q is positive in a South-North direction contrary to s which is positive in a North-South direction.

By the change (2) and under a constant increment Δq, the coordinate q yields too many points at the great altitudes of the magnetic field line and an insufficient number of points at the altitides of the *F*-region. To obtain a more accurate numerical solution of the model equation, the points along the field line must be close together in the *F*-region, while at greater altitudes, the points can be at larger distances from one another.

For the purpose, the following substitution is proposed:

(4) $$x=q^{\alpha}q^{-\alpha}_{max},$$

where $\alpha=2\nu+1$ and ν is a suitably chosen positive whole number. The maximum positive value of q is denoted by q_{max} and is obtained from

(5) $$q_{max}=\frac{r_0^2}{r_b}\left(1-\frac{r_b}{r_0 p}\right)^{1/2},$$

where r_b is the value of r at the lower boundary in the North hemisphere.

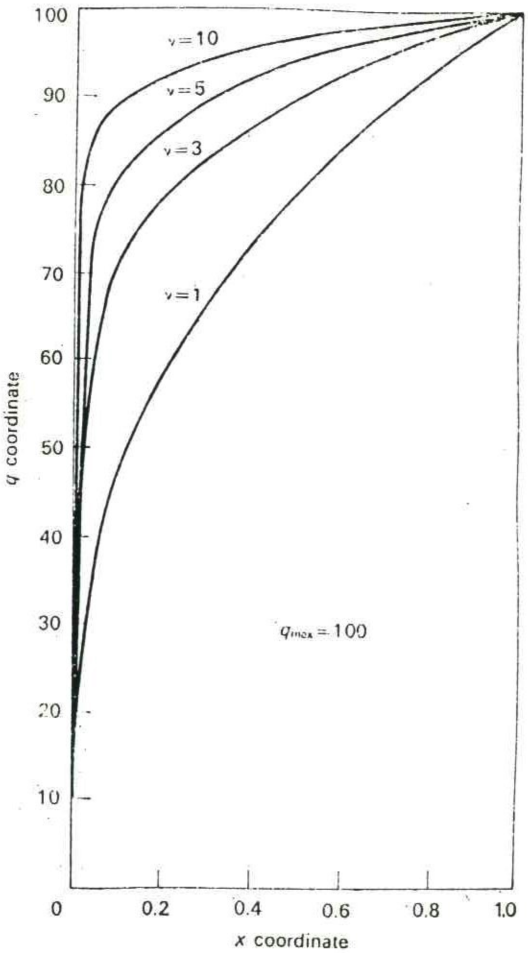

Fig.

From (4) it follows that

(6) $$\frac{\partial}{\partial q}=\chi_2\frac{\partial}{\partial x}$$
$$\frac{\partial^2}{\partial q^2}=\chi_2^2\frac{\partial^2}{\partial x^2}+\chi_1\frac{\partial}{\partial x},$$

where

(7) $$\chi_1=\frac{\alpha(\alpha-1)}{q^2}x,$$
$$\chi_2=\frac{\alpha}{q}x.$$

Besides,

(8)
$$\frac{dG}{dt}=\frac{dG}{dt}+\frac{dG}{dx}\left(\frac{dx}{dt}\right)_q,$$

where

(9)
$$\left(\frac{dx}{dt}\right)_q=-\frac{\alpha r_0^2 v^{\perp}}{2p^2 r_b^3 q_{\max}^2}x.$$

Using (6), (7), (8) and (9), eq. (3) assumes the form

(10)
$$\frac{dG}{dt}=C_1\frac{\partial^2 G}{\partial x^2}+C_2\frac{\partial G}{\partial x}+C_3G+C_4,$$

where

$$C_1=B_1\chi_2^2$$

$$C_2=B_1\chi_1+B_2\chi_2-\left(\frac{dx}{dt}\right)_q$$

$$C_i=B_i,\quad i=3,\ 4.$$

The integration boundaries of eq. (10) are always the fixed values $x=-1$ and $x=1$; $x=-1$ corresponds to the boundary $r=r_b$ in the South hemisphere, $x=1$ corresponds to the boundary $r=r_b$ in the North hemisphere and $x=0$ is located on the equator. The coordinate x is positive in a South-North direction.

By substitution (4) the number of the points is increased in that part of the magnetic field line which is in the F-region and decreased in the rest of its part. The effect is shown in the Figure for different values of the number ν. The needed thickening degree of the points is obtained by a suitable choice of ν: larger value — greater number of points in the F-region.

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Geophysical Institute
Bulgarian Academy of Sciences
Sofia 1113, Bulgaria