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# ANALYTICAL EXPRESSION OF $O^+$–$H^+$ ION TRANSITION SURFACE FOR USE IN IRI

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## ABSTRACT

A global surface of $O^+$-$H^+$ transition level is constructed, based on published data from OGO-6 , Intercosmos-2, Alouette-1, ISS-b, and TAIYO satellites. This surface covers ±60° dipole latitude, all longitudes, two levels of solar activity, summer and winter solstices, and 00 and 12 hours local time. The surface is used as input data to a mathematical model which calculates transition levels in 5-dimensional space: sunspot number (R), month (M), local time (LT), dipole latitude (DL), and longitude (LONG). This model is based on a generalized multivariable polynomial, using a system of linearly independent functions. Model transition levels are compared with averaged data from AE-E and AE-C, as well as rocket measurements from Vertical-6 and Vertical-10. The obtained analytical expression can be directly used in IRI.

## INTRODUCTION

The altitude at which $O^+$ and $H^+$ ion densities become equal ( the so-called transition level - TL), can be suitable for use in IRI. This is the level where the electron density profile changes its slope and it can serve as a base for finding the relative quantity of $H^+$ and $O^+$ ions in the high ionosphere. Bilitza /1/ has recently suggested a way in which the IRI ion composition can be significantly improved, using upper and lower transition levels. These levels can be easily incorporated in the existing Booker /2/ scheme. Following this idea, a mathematical model is now presented, which gives the upper TL surface necessary for direct use. From here on, a differentiation is made between $O^+$-$H^+$ and $O^+$-$He^+$ transition levels. Usually $He^+$ is the minor ion, however there is theoretical and empirical evidence /3,4/ that sometimes $He^+$ rises above $H^+$ in the region of the so-called winter nighttime $He^+$ bulge. The physical processes controlling the density of $H^+$ and $He^+$ are quite different, however, both ions are strongly coupled with their corresponding neutral densities. That is why the focus here will be on $O^+$-$H^+$ transition level behaviour; $O^+$-$He^+$ TLs will be considered in a different paper. An effort is made for combining the already published data for TL variations at different solar and seasonal condition in an analytical expression, thus making it directly useful for IRI. The definition of the data base is discussed first, then the mathematical approach is briefly presented. In the end, the results obtained are compared with the results of other published data.

## DATA BASE

Two types of satellite data are used:

a) Direct measurements: when $H^+$ and $O^+$ densities are continuously measured along the satellite orbit and TL is encountered.

b) Indirect data set: the TL is obtained through calculations from measurements not conducted at TL altitude.

For the purpose of this analysis, published results are used. They contain graphical presentation of generalized TL variations and not incident measured data. These TL values are to a certain extent averages over longitude, local time, month, or solar activity. This fact causes a significant uncertainty when the data have to be disturbed along the parameters used by IRI: sunspot number (R), month (M), local time (H), dipole latitude (DL), and longitude (LONG). The data used are fragmentarily allocated within the space of these parameters. For the whole 5-dimensional surface to be constructed, it was necessary to accept a certain type of daily, seasonal, and cyclic TL variations. It was accepted that these relations are similar to the variations of the total electron content (TEC) obtained in Graz, Austria /5/. Table 1 (in the APPENDIX) shows TL on two levels of solar activity (R=50,100), December and June solstice, for 00 LT and 12 LT. The table contains TL between -60° and +60° dipole latitude and longitude cross-sections every 60°. TL at high solar activity is obtained from OGO-6 /6/ and Intercosmos-2 /7/ data. Both satellites sample the nighttime TL surface almost evenly along the longitudes. Daytime measurements are carried out by OGO-6 at 18 LT, so the values at 12 LT in Table 1 are adjusted for the corresponding zenith angle. The ISS-b satellite has also conducted measurements at high solar activity /8/, but it has rarely crossed the TL surface. A special study of the diurnal equatorial TL variations using data from this satellite is done in /9/. The TL values here are obtained through theoretical approximations of the measured percentage of ion species at about 1100 km. These variations of the TL at the equator are taken into account, except for those of the December solstice, where it was found that the nighttime values are higher than the daytime ones. Direct measurements from AE-E /10/ do not confirm this fact. At low solar activity, TL in Table1 is compiled from the results of Alouette-1 /11/ and TAIYO /12/. The TL values obtained in /11/ are higher than those in /12/, which is probably the result of the different level of solar activity. Table 1 uses TL values that are between those of the two data sources.

The adopted TL surface has in general the following features: There is a broad daytime minimum at the equator, ranging between 960 km at R=50 and 1300 km at R=100. At higher latitudes the TL rises, which is due to the increased $O^+$ density towards the crest and the decreasing $H^+$ density when the outer plasmasphere flux tubes are approached. At nighttime, a local maximum is formed at the equator and minimums at ±30° DL. At low solar activity (R=50) the equatorial TL is 760 km, and the winter minimum is about 660 km. At high solar activity (R=100) the equatorial TL is between 940 and 1000 km, the winter minimum being at 700 km. The seasonal variation is very weak, except at midlatitudes. However, the relation with solar activity is much stronger.

## THEORETICAL APPROXIMATION

The TL variations are approximated by a multivariable polynomial. The fitting generalized polynomial has the form:

$$P(C,N;x)=\sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}\sum_{i_3=1}^{n_3}\sum_{i_4=1}^{n_4}\sum_{i_5=1}^{n_5}c(i_1,\dots,i_5)g_1(i_1,x_1)\dots g_5(i_5,x_5)$$

where $\{g_m(i_m,x_m)\}_{i_m=1}^{n_m}$ is the system of linearly independent functions on the domain of the $m^{\text{th}}$ parameter. The following systems are used:

| | |
|---|---|
| $1, x, x^2, \dots, x^{nm}$ | algebraic basis |
| $1, \sin x, \cos x, \dots, \sin n_m x, \cos n_m x$ | trigonometric basis |
| $\cos((n-1)\arccos x),\ n=1,2,\dots,n_m$ | Tchebishev's basis |

Also, $N=(n_1, \dots, n_5)$, where $n_m$ is the number of functions in the system for the $m^{\text{th}}$ interval and $C=\{c(i_1, \dots, i_5),\ i_m=1, \dots, n_m\}$ is the set of coefficients. The method of weighted least-squares fit is applied for determining the coefficients.

## RESULTS AND THEIR COMPARISON WITH MEASURED TLs

Fig.1 presents the diurnal TL variations at different seasons and 0° longitude. For R=50 the June and December equatorial variations are close, while those at 30° DL differ from each other by 100 km. At R=100, the equatorial TLs are similar; the noon values are about 1150 km and the midnight values are 1000 and 940 km for June and December respectively. More significant is the seasonal difference at DL=30°. The TL amplitude at the summer side is 300 km, while in the winter hemisphere it is 450 km. Fig.2 shows the relations of TL and R. In the model these relations are taken as linear in order to avoid the possibility of large deviations in the end of the interval, if R is expressed as a higher order polynomial. This leads to certain errors, as in the case of Table 2, but they are limited. In all cases presented, TL changes with 500-600 km as R varies from 10 to 150. The TL in the winter nighttime minimum at midlatitudes is an exception. In this case the TL change does not exceed 140 km within the whole range of R. This fact

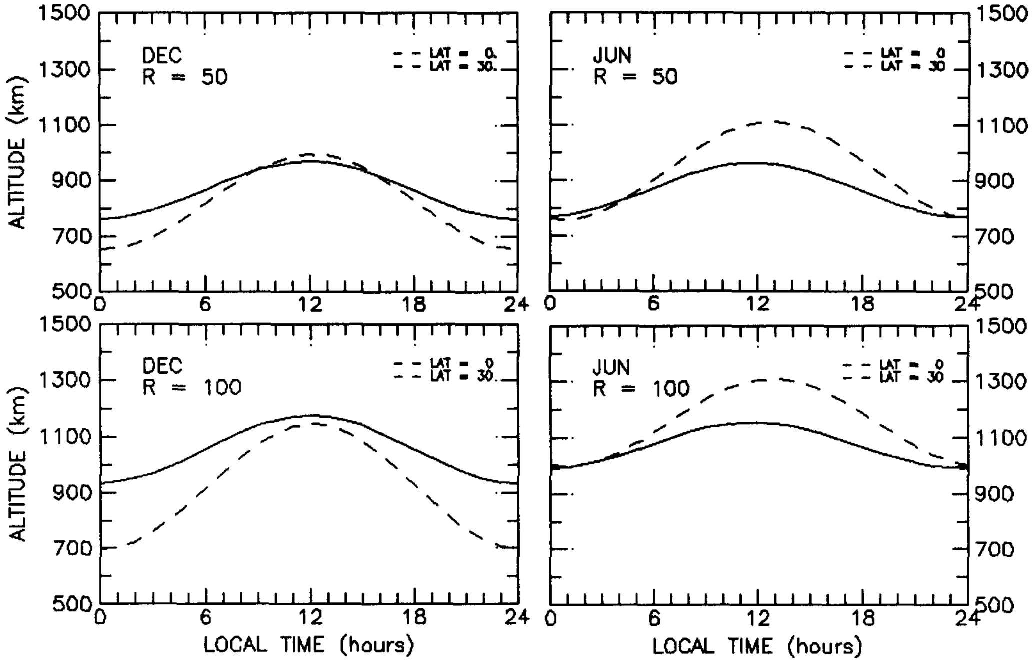

Fig. 1. Diurnal variations of TL at low (R=50) and high (R=100) solar activity for December and June months and O° and 30° latitudes.

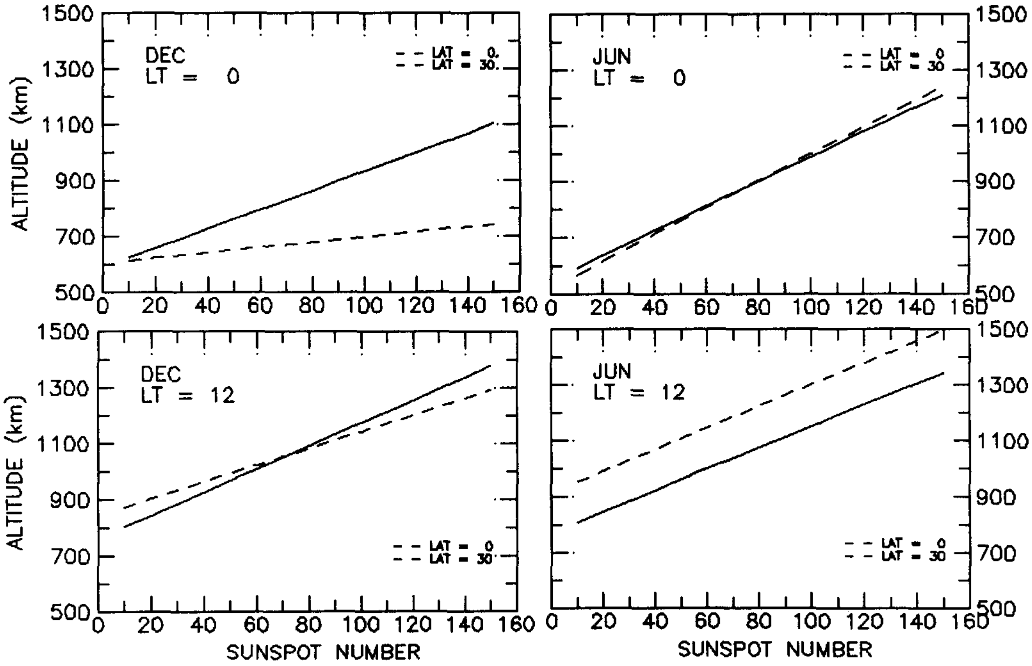

Fig. 2. TL dependance on sunspot number for December and June months and O° and 30° latitudes.

reflects the collapse of the nighttime midlatitude F-region, when the recombination cannot be compensated by plasmaspheric $H^+$ flow because of the low charge exchange rate with $O^+$.

Several new sources of measured TL values have been used for comparison with the model calculations. Equatorial (±7.5 DL) profiles of $O^+$, $H^+$, and $He^+$, averaged over December, June, and the equinoxes are shown in /10/. The corresponding TLs are presented in the column before the last in Table 2. The last column contains the TL values calculated by the above method for the corresponding conditions. The model TL values for the three seasons are higher than the measured values which is due to the chosen linear relation of the TL with R. This relation is determined by the TL at R=50 and R=100 and will naturally give maximum deviation in the end of the interval, at R=10. In /13/ AE-C ion profiles are presented. They are obtained in May 1975 and taken as an average between 20° and 40° DL. Table 2 also contains two rocket measurements, conducted by Vertical-6 and Vertical-10 /14/. The latitude pattern of the TL has a sharp gradient around 40°, that is why the last three measurements deviate significantly from the model results. This is evident in the case of Vertical-6.

TABLE 2 Comparison between measured and model TL values

| | Source | Season<br>Time of measurement | R | TL measurement<br>night / day | TL model |
|---|---|---|---|---|---|
| 1 | AE-E /6/ (equator) | 1975-1976 Dec. solstice | 10 | 580 / 820 | 627 / 804 |
| 2 | AE-E /6/ (equator) | Equinoxes | 20 | 590 / 800 | 649 / 846 |
| 3 | AE-E /6/ (equator) | June solstice | 10 | 536 / 790 | 593 / 807 |
| 4 | AE-C /9/ (20°-40° DL) | May 1974 | 40 | / 1075 | /1047 |
| 5 | Vertical 6 /10/ (40° DL) | 25 Oct 1977 15:25 LT | 44 | 1200 | 1020 |
| 6 | AE-C /9/ (40° DL) | 23 Dec 1981 23:00 LT | 150 | 735 | 734 |

## DISCUSSION

A mathematical model is developed, which approximates TL in the 5-dimensional space of: sunspot numbers, months, hours, latitudes, and longitudes. A table is constructed containing TL values adapted from published TL variations data at different conditions. The basic data shown in Table 1, is used as an input array to the model. The TL values have been interpolated and certain relationships of the daily, seasonal, and cyclic variations are accepted. The TL obtained values are then compared to seasonally averaged satellite measurements and individual rocket soundings. The comparison shows a relatively good consistency, having in mind the significant differences in published TL data. This model allows a future improvement in the TL model, when individual measurements from the satellites are included, replacing the now-used synthetic TL surface.

## ACKNOWLEDGEMENTS

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# APPENDIX H – ANALYTICAL EXPRESSION OF $O^+/H^+$

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***Table 1*** *$O^+$-$H^+$ Transition Level in km*

Dec. solstice 00:00 LT (R=50)

| | | | Long | | | |
|---|---|---|---|---|---|---|
| MLat | 0 | 60 | 120 | 180 | 240 | 300 |
| 60 | 754 | 754 | 754 | 754 | 754 | 754 |
| 50 | 740 | 735 | 741 | 741 | 751 | 745 |
| 40 | 705 | 720 | 724 | 720 | 705 | 705 |
| 30 | 645 | 650 | 653 | 646 | 645 | 645 |
| 20 | 625 | 627 | 628 | 632 | 625 | 625 |
| 10 | 715 | 715 | 715 | 713 | 711 | 709 |
| 0 | 765 | 765 | 765 | 765 | 765 | 765 |
| -10 | 750 | 750 | 750 | 750 | 754 | 750 |
| -20 | 724 | 722 | 720 | 724 | 728 | 724 |
| -30 | 715 | 713 | 710 | 709 | 702 | 709 |
| -40 | 730 | 735 | 739 | 739 | 725 | 730 |
| -50 | 775 | 781 | 785 | 790 | 775 | 780 |
| -60 | 824 | 824 | 824 | 824 | 824 | 824 |

June solstice 00:00 LT (R=50)

| | | | Long | | | |
|---|---|---|---|---|---|---|
| MLat | 0 | 60 | 120 | 180 | 240 | 300 |
| 60 | 824 | 824 | 824 | 824 | 824 | 824 |
| 50 | 765 | 770 | 765 | 771 | 775 | 780 |
| 40 | 725 | 730 | 730 | 735 | 739 | 739 |
| 30 | 714 | 726 | 732 | 730 | 727 | 726 |
| 20 | 718 | 714 | 714 | 712 | 710 | 714 |
| 10 | 749 | 745 | 745 | 745 | 745 | 745 |
| 0 | 766 | 762 | 762 | 762 | 762 | 762 |
| -10 | 750 | 748 | 754 | 754 | 754 | 752 |
| -20 | 690 | 690 | 690 | 692 | 693 | 697 |
| -30 | 635 | 635 | 635 | 640 | 643 | 636 |
| -40 | 670 | 670 | 670 | 685 | 689 | 685 |
| -50 | 725 | 719 | 716 | 709 | 715 | 715 |
| -60 | 754 | 754 | 754 | 754 | 754 | 754 |

Dec. solstice 00:00 LT (R=100)

| | | | Long | | | |
|---|---|---|---|---|---|---|
| MLat | 0 | 60 | 120 | 180 | 240 | 300 |
| 60 | 910 | 910 | 910 | 910 | 910 | 910 |
| 50 | 880 | 900 | 900 | 908 | 916 | 917 |
| 40 | 721 | 729 | 731 | 736 | 736 | 738 |
| 30 | 672 | 679 | 682 | 685 | 685 | 689 |
| 20 | 667 | 660 | 660 | 653 | 653 | 660 |
| 10 | 687 | 680 | 680 | 680 | 680 | 680 |
| 0 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| -10 | 1080 | 1090 | 1090 | 1090 | 1090 | 1090 |
| -20 | 1050 | 1050 | 1050 | 1070 | 1070 | 1070 |
| -30 | 1000 | 1000 | 1000 | 1020 | 1030 | 1030 |
| -40 | 970 | 970 | 970 | 1030 | 1040 | 1040 |
| -50 | 1060 | 1060 | 1060 | 1070 | 1080 | 1080 |
| -60 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 |

June solstice 00:00 LT (R=100)

| | | | Long | | | |
|---|---|---|---|---|---|---|
| MLat | 0 | 60 | 120 | 180 | 240 | 300 |
| 60 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 |
| 50 | 922 | 912 | 902 | 922 | 942 | 932 |
| 40 | 950 | 940 | 930 | 940 | 950 | 950 |
| 30 | 1011 | 991 | 981 | 976 | 971 | 991 |
| 20 | 1040 | 1030 | 1030 | 1025 | 1020 | 1030 |
| 10 | 1049 | 1039 | 1039 | 1039 | 1039 | 1039 |
| 0 | 1050 | 1040 | 1040 | 1040 | 1040 | 1040 |
| -10 | 1028 | 978 | 948 | 948 | 948 | 988 |
| -20 | 686 | 686 | 686 | 726 | 766 | 726 |
| -30 | 660 | 660 | 660 | 695 | 730 | 690 |
| -40 | 731 | 731 | 731 | 811 | 881 | 811 |
| -50 | 998 | 898 | 848 | 938 | 998 | 998 |
| -60 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 |

12:00 LT

| | R=100 | R=100 | R=50 | R=50 |
|---|---|---|---|---|
| MLat | JUN | DEC | JUN | DEC |
| 60 | 1430 | 1363 | 1200 | 1100 |
| 50 | 1420 | 1293 | 1189 | 1092 |
| 40 | 1380 | 1224 | 1152 | 1061 |
| 30 | 1300 | 1200 | 1085 | 1002 |
| 20 | 1250 | 1190 | 995 | 973 |
| 10 | 1200 | 1170 | 950 | 948 |
| 0 | 1150 | 1150 | 940 | 936 |
| -10 | 1170 | 1200 | 949 | 945 |
| -20 | 1180 | 1250 | 978 | 962 |
| -30 | 1200 | 1330 | 1014 | 1011 |
| -40 | 1220 | 1380 | 1070 | 1100 |
| -50 | 1293 | 1420 | 1095 | 1175 |
| -60 | 1363 | 1430 | 1100 | 1200 |