# Ionospheric Electron Content model based on a steady-state theoretical profiler: II. Model evaluation

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**Abstract**. Reported is an evaluation study of a newly proposed Ionospheric Electron Content (IEC) model. This model employs a steady-state theoretical code to simulate the topside ion and electron density distribution. The evaluation procedure is based on a long-term series of IEC data using the Differential Doppler effect on signals from the U.S. Navy Navigation Satellite System (NNSS). Extensive comparison between actual NNSS data and modelled IEC values are presented for the most important diurnal, latitudinal, seasonal, and solar-activity variations. The following points are among the major findings: (1) The steady-state theoretical profiler is a high-precision tool when additional measurements are available such as NmF2 and $O^+$-$H^+$ transition level; (2) The IEC model is much more suitable for applications at low and middle latitudes; (3) A comparison with the International Reference Ionosphere (IRI) empirical model values reveals that in many cases the presented IEC model is a better option to use.

**Keywords:** ionospheric electron content, mathematical model, evaluation

## 1. Introduction

In the previous companion publication (Stankov, 2001), a new Ionospheric Electron Content (IEC) model has been formulated. The general idea behind this model is to produce a vertical electron density profile of high quality and then to integrate it numerically in order to derive the electron content up to a given ceiling height. Because the major contribution comes from the ionosphere, and because of the large time-series data have been accumulated via the U.S. Navy Navigation Satellite System (NNSS), the emphasis here is on the ionospheric rather than the total electron content. The construction of the IEC model is built upon a previously developed steady-state theoretical profiler (Stankov, 1994; Stankov, 1996), tied to empirically obtained values of the O+-H+ transition level and ionosonde data.

The purpose of this publication is to present an evaluation of the above model of ionospheric content with actual measurements for various spatial and temporal conditions. For this purpose, data from NNSS satellites have been used, for both low and high solar activity conditions and various latitudes at the sub-ionospheric point (from 40°N to 60°N). The IEC model requires several inputs, as for example the neutral atmosphere parameters, which are obtained from satellite measurements or the empirical model MSIS.

The paper is organised in the following way. First, details are provided on the data base used for comparison between empirical and modelled values. Next, the evaluation procedure is outlined with the error estimates used. Finally, the comparison is made and discussed.

## 2. Data base

It is not possible to gain both temporal and spatial dependencies with one receiving station and with one satellite, like in the case of geostaniary satellites. Although the GPS-based measurements offer good opportunity for wourld-wide coverage, the NNSS is an established system providing reliable data for decades. It gives the latitude dependence of electron content for essentially constant time (the typical duration of a high-elevation NNSS pass is 20 minutes). The NNSS satellites have nearly circular and nearly polar orbits in about 1100 km height. The data are gained by means of the Differential Doppler Effect (Carrier Phase Difference) on the signals from NNSS.

The IEC database used in this study has been created (Feichter and Leitinger, 1993) from measurements acquired at two receiving stations – Lindau / Harz (10.1°E, 51.6°N) and Graz (15.5°, 47.1°N). The Graz station data has been used for ‘calibration’ purposes by the ‘two-station’ method (Leitinger et al., 1975). Reliable and long time-series data are available for a sequence of equidistant geographic latitudes in the range 40°-60°N for both slant and vertical values. Vertical values will be used here for the latitudes 40, 45°, 50°, 55°, and 60°N. NNSS observations, spanning more than a solar activity cycle, have been used to create two sets of data - a low-solar activity (LSA) data set (for sunspot number $R<40$) from 1975-1976 and a High Solar Activity (HSA) data set ($130<R<170$) assembled during the 1978-1982 period.

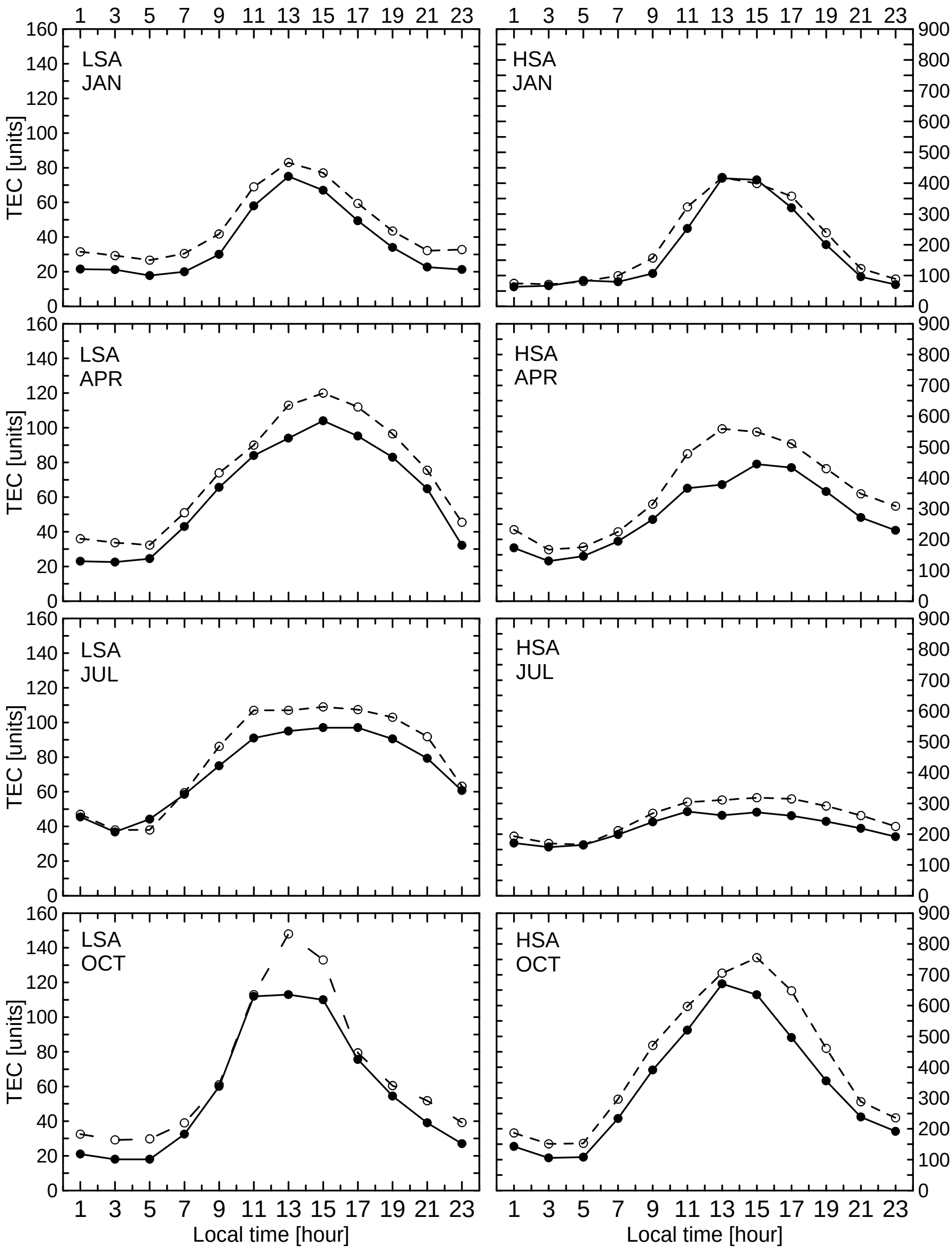


**Fig.1** Diurnal curves (monthly medians) of the ionospheric electron content in units of $10^{15}m^{-2}$ derived from the Differential Doppler effect on signals from NNSS. Receiving station Lindau (10.1°N, 51.6°E). Sub-ionospheric points at 45°N(dashed line), 55°N(solid line).

The electron content demonstrates strong variability which is unusual for an integral quantity (**Fig.1**). It depends on the location (spatial variation), local time (diurnal variation), time in the year (annual variation), solar activity (solar cycle variation).

The IEC *diurnal* behaviour is similar for LSA and HSA. During winter, the electron content increases sharply in the morning hours, reaches its maximum around noon, and then decreases slowly toward the night values. During summer, there is a retardation of the peaks and the second-half day components. The HSA summer maxima are much lower than the winter and equinox ones. Notice also the high late-evening values in July - this is because the F-layer moves upwards during afternoons leading to an increase of the ionisation in the topside of the F-layer and the content values are preserved unchanged until 18:00-20:00 LT. Combined F2-layer maximum density and total electron content data (in middle latitudes) show that, while the winter-day values are higher than the summer-day values, the highest values occur at one or the other of the equinoxes.

A longterm study of the *annual* behaviour reveals distinct differences between day-time and night-time characteristics. During day-time, two maxima occur around the equinoxes (April/October). Reported is also a 22-year periodicity in the equinoctal peak - in 1975-76 minimum the autumn peak is higher than the spring peak, while in 1986 it is just the opposite. During night, for both LSA and HSA, only one maximum is observed (in June) while the minimum is in the winter (December/January).

Generally, the TEC decreases with increasing latitude both during LSA and HSA and both during day and night. The differences are large, especially in the equinoctial months - the peaks increase by 50% at LSA and approximately 100% at HSA (spring equinox). The mentioned changes are not so pronounced during winter and summer seasons. The occurrence of the maxima is also latitude dependent in such a way that it happens earlier at higher latitudes.

## 3. Evaluation procedure

The model is run for various field lines and electron concentration and is afterwards integrated along the intersections of the vertical path with the field lines. Detailed description of the model geometry, numerical callculations and input parameters has been already provided (Stankov, 2001).

After obtaining the measurement data and model calculations, the results for the corresponding spatial and temporal conditions are compared in the ‘climate’ sense, i.e. for monthly median values. The estimation is based on the relative (percentage) deviations of the model value from the measurements, i.e.

$$R = \left(IEC_{model} - IEC_{measured}\right)/\, IEC_{measured}$$

The absolute value of this error, and its mean at various time and space intervals are also used to evaluate the model performance under different conditions. The IEC model results are compared also with the International Reference Ionosphere (IRI) model calculations of the ionospheric electron content.

To facilitate the comparison, all measurements were subjected to Fourier analysis and a reconstruction formula was developed (Feichter and Leitinger, 1993) to deduce the content at an arbitrary local time and day number:

$$IEC(t,d) = \sum_{i=0}^{11}\sum_{j=0}^{11} C_{ij}\left[\begin{matrix}\cos\\ \sin\end{matrix}\left(m\frac{2\pi}{24}t\right)\right]\left[\begin{matrix}\cos\\ \sin\end{matrix}\left(m\frac{2\pi}{365}d\right)\right]$$

where $t$ – local time [hours], $d$ – day number-15 [days] $m=i$, $n=j$ for $i,j=0,1,...,6$ and $m=i-6$, $n=j-6$ for $i,j=7,8,...,11$. The formula coefficients are given in **Table1**, where $i$ - column index {0(a0),...,11(b5)} and $j$ - row index {0(A0),...,11(B5)}.

| 45°N | a0 | a1 | a2 | a3 | a4 | a5 | a6 | b1 | b2 | b3 | b4 | b5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A0 | 287.5 | -7.32 | -72.70 | -22.42 | 18.87 | 5.01 | -5.94 | 15.54 | 8.95 | 26.34 | -2.81 | -16.34 |
| A1 | -156.1 | -64.01 | 44.06 | 12.03 | -11.70 | 2.53 | 8.94 | 35.40 | -2.92 | -22.47 | 5.18 | 3.57 |
| A2 | 34.52 | 27.58 | -5.57 | -2.45 | -6.14 | -6.10 | -2.51 | -16.38 | -3.80 | -1.43 | -4.99 | 14.03 |
| A3 | -6.69 | -8.20 | 1.14 | -1.56 | 12.49 | 3.86 | -6.46 | 3.18 | -2.38 | 9.12 | -0.37 | -9.50 |
| A4 | 4.16 | -1.50 | -3.12 | 3.57 | -2.19 | 1.04 | 3.42 | -4.67 | 5.07 | -1.15 | -3.56 | 5.22 |
| A5 | -0.37 | 6.05 | 3.01 | -4.55 | -1.04 | -2.77 | -2.62 | 4.59 | -6.02 | -1.10 | 4.86 | -3.94 |
| A6 | -0.96 | -3.65 | -1.31 | 2.38 | -0.70 | 1.23 | 2.30 | -1.17 | 3.42 | -0.55 | -1.64 | 1.73 |
| B1 | -74.64 | -13.96 | 31.13 | -2.48 | -1.20 | 3.22 | -8.05 | -11.32 | -21.88 | -1.10 | 1.30 | 4.05 |
| B2 | 6.88 | 12.76 | -3.58 | 11.07 | 0.27 | -2.67 | 5.70 | -7.37 | 1.50 | 2.57 | -3.13 | -1.79 |
| B3 | 2.93 | 7.91 | 5.75 | -9.93 | -4.08 | 4.90 | 0.12 | 4.16 | 1.66 | -3.27 | 4.17 | -1.65 |
| B4 | 0.66 | -10.29 | -3.36 | 6.64 | -6.28 | -4.19 | 2.18 | -2.49 | -2.47 | 5.37 | -2.34 | -1.21 |
| B5 | 2.43 | 2.96 | 1.36 | -3.32 | -1.36 | 1.60 | 0.20 | 1.63 | 0.99 | -1.09 | 1.39 | -0.64 |
| A0 | 67.21 | -18.39 | -2.64 | 1.08 | -0.34 | 0.62 | -1.15 | 5.29 | -0.32 | 0.27 | -1.03 | -2.40 |
| A1 | -32.29 | 2.54 | 7.63 | 1.48 | -3.74 | -0.18 | 0.35 | 1.34 | -2.32 | -2.23 | 0.94 | 2.21 |
| A2 | 5.64 | 6.17 | -3.11 | -0.07 | 1.83 | 0.24 | 0.06 | -5.55 | -0.00 | 3.89 | 0.45 | -1.22 |
| A3 | -1.17 | -0.50 | 1.79 | 0.21 | -1.37 | -0.79 | 0.30 | 1.72 | 0.80 | -1.47 | -1.92 | 1.64 |
| A4 | 0.96 | 0.09 | -0.92 | -0.14 | -0.24 | 0.74 | 0.25 | 0.04 | 0.78 | -0.14 | -0.20 | -0.75 |
| A5 | -0.29 | 0.00 | 0.50 | 0.10 | 0.86 | -0.73 | -0.44 | -0.66 | -1.29 | 0.74 | 0.90 | 0.42 |
| A6 | 0.01 | -0.56 | -0.24 | -0.08 | -0.33 | 0.42 | 0.14 | 0.58 | 0.47 | -0.48 | -0.16 | -0.33 |
| B1 | -15.95 | 8.11 | 2.23 | -1.25 | 0.57 | 0.07 | -0.26 | -4.30 | -0.26 | 0.96 | -1.11 | -0.18 |
| B2 | -1.60 | 6.55 | -2.17 | -0.40 | 0.89 | -0.93 | 0.05 | 0.09 | 0.66 | -0.42 | 0.33 | -0.04 |
| B3 | 1.39 | -0.23 | 0.79 | -0.97 | -0.85 | -0.04 | 0.16 | 0.23 | -0.06 | -0.66 | 0.75 | 0.12 |
| B4 | 0.71 | -2.17 | -0.23 | -0.07 | 0.69 | 1.01 | -1.04 | 0.71 | 0.61 | -0.32 | 0.56 | -0.32 |
| B5 | 0.78 | -0.23 | 0.22 | -0.30 | -0.30 | -0.01 | 0.06 | 0.16 | -0.02 | -0.24 | 0.28 | 0.04 |

**Table 1.** Fourier coefficients: horizontally – diurnal coefficients, vertically – annual coefficients for HSA (top) and LSA (bottom); NNSS – based IEC at 45°N; station Lindau (10.1°N, 51.6°E).

## 4. Results and discussion

Using the steady-state model, the electron densities (and the integrated IEC) have been calculated for conditions closely matching those during the NNSS-based data acquisition. Here, the neutral atmosphere parameters were taken from the AE-C satellite measurements for low-solar activity conditions and MSIS-90E empirical model was used for the HSA values. The electron and temperatire initial/boundary values were taken from the IRI-95 model. The O+ ion inital density is determined with the searching procedure (Stankov, 1996) and the topside profile fitted to the upper transition level values (Stankov, 2001).

Previous evaluation studies reveal that the semi-empirical approach has sufficient advantages over the purely theoretical (Brown et al., 1991) simulations. This is confirmed by the results presented here.

### 4.1 Diurnal behaviour

The results of the reconstructed diurnal behaviour is presented in **Fig.2** with the help of the relative deviations of IEC from the measured values. Overall, the simulation is obviously better during the mid-day and midnight hours, than it is in the remaining hours. That is why, the dawn-dusk (DD) periods ( 04:00-08:00LT and 16:00-20:00LT) are separately estimated from the twenty-four-hours (24H) period (00:00-24:00LT). During LSA, the absolute value of the mean relative error for the twenty-four-hours period varies between 7.2 % (in April) and 10.7 % (in January), while the corresponding dawn-dusk period, the average deviations vary between 7.4% and 17.4%. During HSA conditions, the behaviour is essentially preserved – the all-day averages vary between 7.5% and 13.5%, while during the dawn/dusk time the errors are between 8.2 and 21.1%. It is obviously that all errors tend to increase with rising solar activity. Generally, the model systematically underestimates the HSA diurnal behaviour. One possible reason can be the MSIS neutral atmosphere, which is supported by the fact that the LSA simulations are better by few percents. Another reason is the inaccuracy of the upper transition level data, particularly because the HSA values are obtained via extrapolation from those at medium solar activity (R=100).

The relatively large values during the dawn-dusk period are expected due to the steady-state nature of the theoretical profiler. Unless correct input values (via in-situ and ionosonde measurements) are used, this model has no ambition of 'perfectly' simulating these sunrise and sunset periods of quick changes. However, the simulation was improved drastically after utilising the values from the upper transition values.

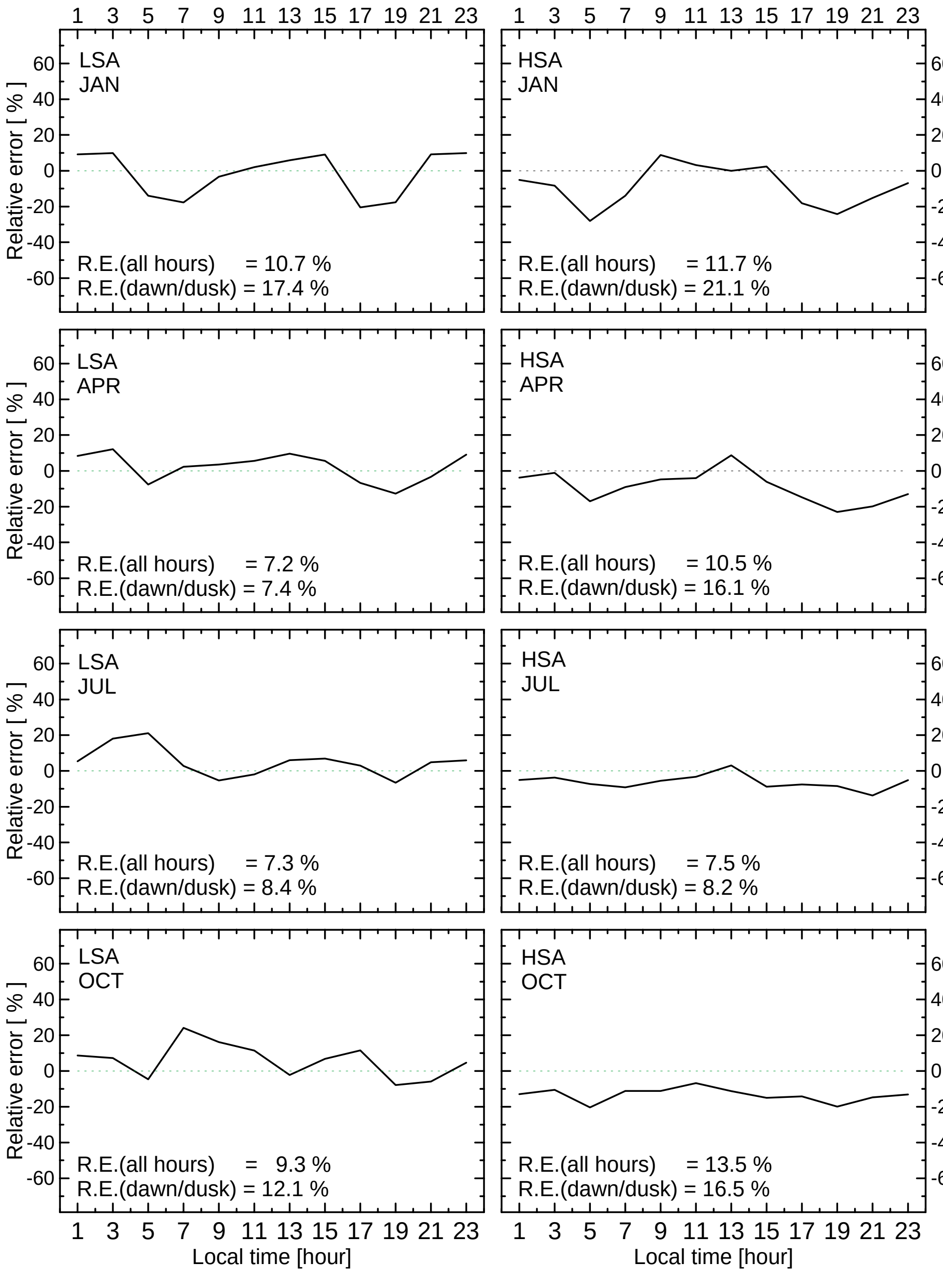


**Fig.2** Diurnal variations of the relative error (TECmod-TECmes)/TECmes at 45°N (solid line) for low solar activity (left-hand panels) and high solar activity (right-hand panels). Root-mean-square and average relative errors are also provided in the bottom left corner of each panel.

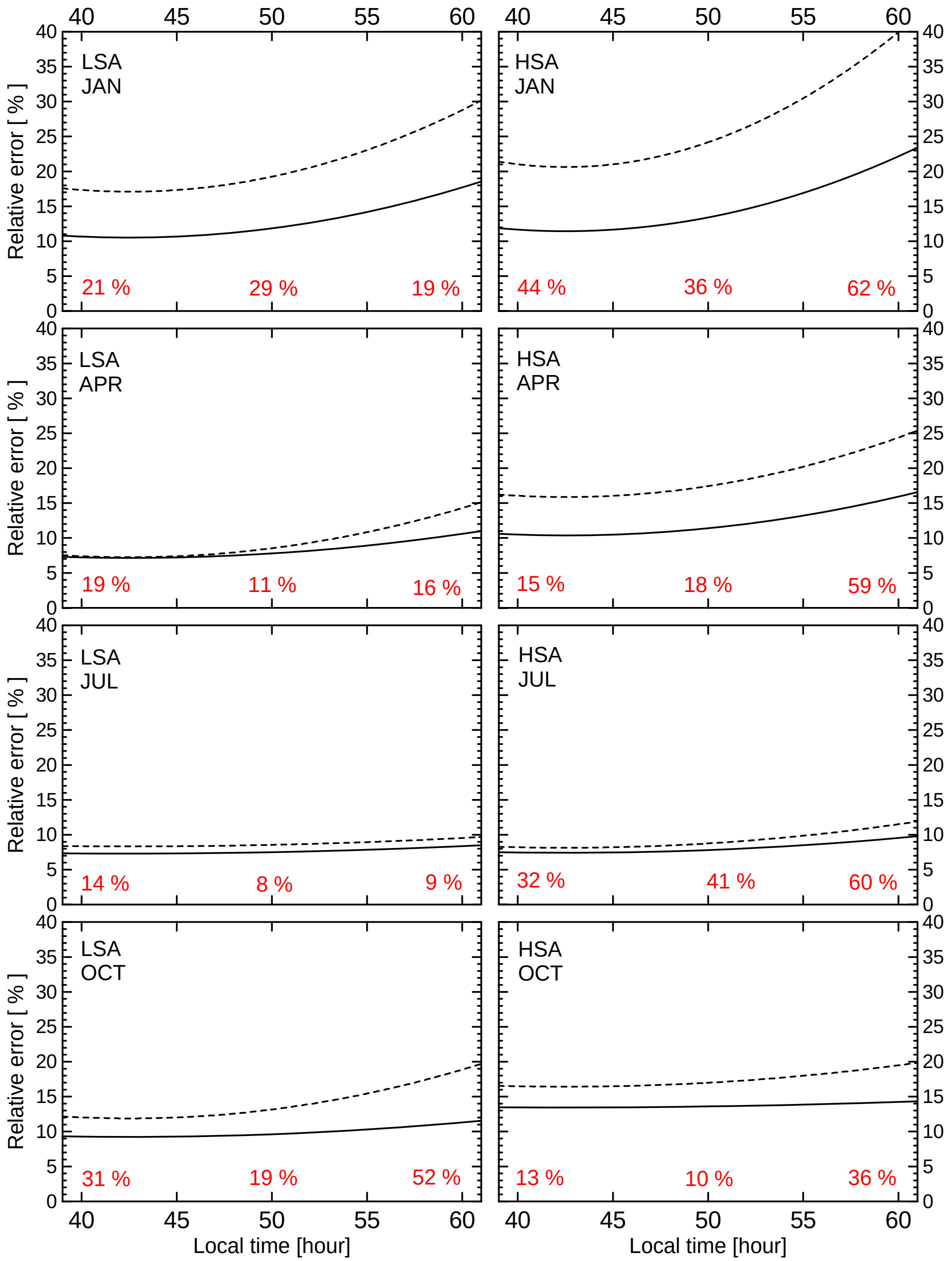


**Fig.3** Latitudinal variations (approximated) of the average relative errors for the 24-hour period (solid line) and for the dawn/dusk period (dashed line). The IRI-95 percentage deviations are provided for 40°N, 50°N, and 60°N.

### 4.2 Latitudinal behaviour

In order to better investigate the abilities to model the IEC latitudinal variations, the IEC model has been run for several field lines so the vertical quantities of the IEC has been obtained at different latitudes in the range 40-60°N. These model calculations have been compared with the corresponding measurements and graphically presented in **Fig.3** for LSA (left-hand side) and HSA (right-hand side). The DD periods are again presented separately (dashed lines) from the 24H based estimations (solid lines).

In general, all errors increase at higher latitudes. It is explained with the nature of the theoretical profiler (which is designed for low and middle geomagnetic latitudes) and with the ion through effects which is difficult to model with the proposed approach. Worst are the results for January when at HSA errors can reach 40% at 60°N. The large errors are attributed mostly to inaccuracies for the DD periods (notice the large gap between the two curves). The best results are obtained during summer. Then the relative deviations do not exceed 10-12% on average. The gap between the DD and 24H average curves is small.

### 4.3 Comparison with IRI

The IRI-95 empirical model has been run for day 16 of January, April, July, October and with the average solar activity values observed during the NNSS measurements (Feichter and Leitinger, 1993). The following IRI options have been used: URSI maps for the peak density NmF2, CCIR maps for the peak height HmF2, and Danilov-95 model for the ion composition. The IEC is integrated from 50 to 1100 km with 5 km height steps. Percentage deviations, based on averages at 0100, 0700, 1300, and 1900 LT, are given in **Fig.3** for 40°N, 50°N, and 60°N latitudes. Generally, the IRI tend to significantly overestimate the IEC values, particularly during HSA. Also, the same trend of increasing the error at higher latitudes is observed during HSA.

In many cases the IEC simulates the average electron content much better than the IRI, e.g. during equinox months of LSA and all months of HSA.

## 5. Conclusions

Reported was an evaluation of a newly proposed IEC model formulated in a previous publication (Stankov, 2001). This model employs a steady-state theoretical code to simulate the topside ion and electron density distribution and the IEC is obtained after integrating the vertical electron density profile at a given spatial location and time. The evaluation procedure is based on a long-term series of IEC data using the NNSS-based observations of the electron content up to 1100km altitude. The extensive comparison between actual NNSS data and modelled IEC values yields the following conclusions:

- The steady-state theoretical profiler is a high-precision tool when additional measurements are available such as NmF2 and $O^+$-$H^+$ transition level;
- The IEC model is much more suitable for applications at low and middle latitudes;
- A comparison with the International Reference Ionosphere (IRI) empirical model values reveals that in many cases the presented IEC model is a better option to use.

The constructed model can be run at an operational mode as well and can be further improved with new measurements of the upper transition level, particularly for HSA.

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