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# EVALUATION OF ANALYTICAL IONOSPHERIC MODELS USED IN ELECTRON DENSITY PROFILE RECONSTRUCTION

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New formulae based on the Sech-squared, Exponential and Chapman layers are introduced for a TEC-based electron density reconstruction technique using ionosonde and upper ion transition level data. These analytical ionospheric models are implemented and tested as reconstruction tools with the help of satellite in-situ measurements of the ion and electron densities. Particular attention is drawn to the calculation quality of the scale height in the upper ionosphere, from $h_m$F2 up to the $O^+-H^+$ transition height.

**Keywords:** electron density reconstruction; ionosphere profiler; total electron content

## 1. Introduction

The ground-based sounding measurements alone are incapable of reliably modelling the topside electron density distribution. A novel method for reconstructing the topside electron density profile has been offered (Stankov and Muhtarov 2001). This method is based on ground measurements of the total electron content (TEC) and uses also empirical values of the upper ($O^+-H^+$) ion transition height and ground ionosonde data to construct two-scale height profiles of the electron density. The approach has been already described and demonstrated with the help of the Sech-squared analytical model as a topside density profiler and tested on GPS TEC measurements under various geophysical conditions (Stankov et al. 2002).

Several analytical models of the ionospheric F region are traditionally used for numerical modelling and radio propagation work. Some of them can be very useful for the above profile reconstruction: Exponential, Sech-squared, Chapman, and Parabolic layers. These layers produce different (in shape) top-side density profiles which are shown in Fig. 1 for given scale height (100 km), maximum density ($1.0 \times 10^5$ cm$^{-3}$), and height of the peak density (300 km). No estimation is made so far of how these models prove good as profile reconstruction tools, particularly using GPS TEC.

The research presented here aims at investigating the possibility of using the Exponential and Chapman layers in addition to the Sech-squared layer, and comparing the results with independent data such as satellite in-situ measurements of the ion densities.

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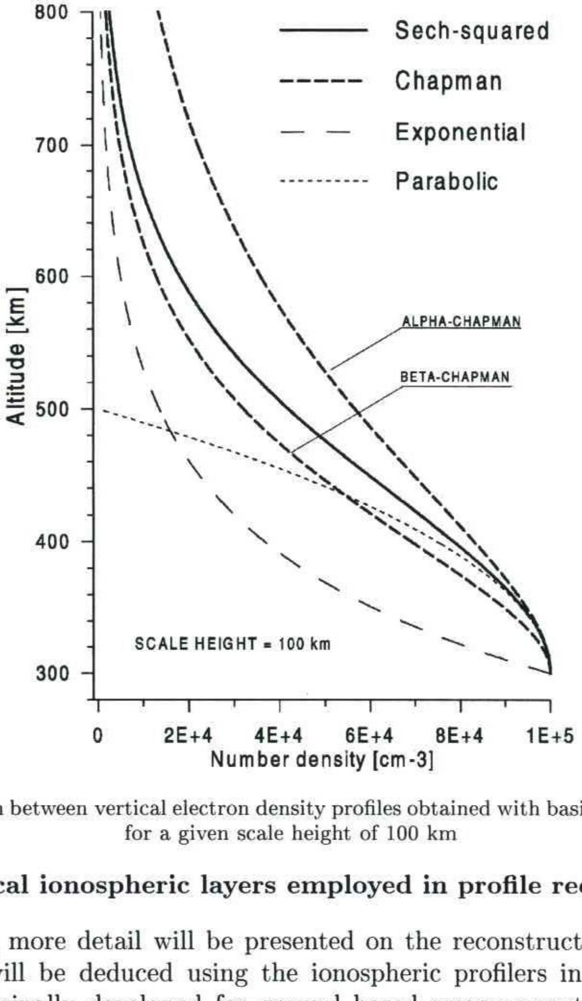


Fig. 1. Comparison between vertical electron density profiles obtained with basic analytical models for a given scale height of 100 km

## 2. Analytical ionospheric layers employed in profile reconstruction

In this part, more detail will be presented on the reconstruction method and new formulae will be deduced using the ionospheric profilers in question. The method was originally developed for ground based measurements of GPS TEC, however it can be extended to space-based TEC measurements as well. The available total electron content is split in two parts — bottomside (below $h_m$F2), and topside (above $h_m$F2). The bottomside electron density profile and its correspondent electron content, $TEC_b$, are calculated from $f_o$F2, M(3000) F2 and $f_o$E using established methods and models (Stankov and Muhtarov 2001, Stankov et al. 2002). The top-side TEC, i.e. $TEC_t$, is calculated as the difference between the GPS-derived TEC and the already calculated bottom-side part of the TEC, that is $TEC_t = TEC - TEC_b$. The topside TEC quantity, together with the $O^+$–$H^+$ transition height and a topside-density distribution formula, are used to construct a transcendental equation of only one unknown variable — the topside $O^+$ ion scale height. The calculated scale height is then used in the chosen profiler to obtain the height distribution of the ion and consequently the electron density distribution. At this stage, $O^+$ and $H^+$ ions are only considered, and quasi-neutrality is assumed.

The top-side electron content is the integral of the electron content from the F2-layer peak density height, $h_m$, up to the upper boundary height $h_c$ (the height of the transmitting satellite or infinity), i.e.

$$TEC(h_m, h_c) = \int_{hm}^{hc} N_e(h)dh = \int_{hm}^{hc} N_{\mathrm{O}^+}(h)dh + \int_{hm}^{hc} N_{\mathrm{H}^+}(h)dh \qquad (1)$$

and both integrals on the right-hand side are solved in the same way, which will be described below for each profiler. The most suitable (in view of numerical handling) analytical layers are the Sech-squared, Exponential, and Chapman layers.

### *2.1 Sech-squared layer*

The sech-squared (Epstein) layer is defined as:

$$N_i(h) = N_i(h_m) sech^2\left(\frac{h - h_m}{2H_i}\right) \qquad (2)$$

where $N_i(h)$ is the ion ($\mathrm{O}^+$ or $\mathrm{H}^+$) density at height $h$, $H_i$ is the ion scale height, and $sech(h) = 1/\cos h(h)$, $\cos h(x) = 0.5(\exp(x) + \exp(-x))$. The integrals in Eq. (1) are solved by applying the following three substitutions:

$$x = \left(\frac{h - h_m}{2H}\right); \ \lim_{h \to h_c} x = \frac{h_c - h_m}{2H}, \ \lim_{h \to h_m} x = 0$$
$$y = 2x; \ \lim_{x \to x_c} y = \frac{h_c - h_m}{H}, \ \lim_{x \to x_m} y = 0$$
$$z = \exp(y); \ \lim_{y \to y_c} z = \exp\left(\frac{h_c - h_m}{H}\right), \ \lim_{y \to y_m} z = 1\,.$$

Thus, for the oxygen ion the solution is found by the following deduction:

$$\int_{hm}^{hc} N(h)dh = HN_m \int_{xm}^{xc} \frac{4e^{2x}}{(1+e^{2x})^2}dx =$$
$$= 4HN_m \int_{ym}^{yc} \frac{e^y}{(1+e^y)^2}dy = 4HN_m \int_{zm}^{zc} \frac{1}{(1+z)^2}dz = 2HN_m$$

for $h_c \gg h_m$, and the topside electron content becomes:

$$TEC_t = 2H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m) + 2H_{\mathrm{H}^+}N_{\mathrm{H}^+}(h_m)\,. \qquad (3)$$

Considering the fact, that at the upper transition level $h_{tr}$ the oxygen and hydrogen ion densities are equal, the transcendental equation will be (Stankov et al. 2002):

$$\left(\frac{16V}{(16V-1)}N_m - \frac{1}{2(16V-1)H_{\mathrm{O}^+}}TEC_t\right) sech^2\left(\frac{h_{tr} - h_m}{2H_{\mathrm{O}^+}}\right) -$$
$$-\left(\frac{1}{2(16V-1)H_{\mathrm{O}^+}}TEC_t - \frac{1}{(16V-1)}N_m\right)\exp\left(-\frac{h_{tr} - h_m}{32VH_{\mathrm{O}^+}}\right) = 0\,.$$

The only unknown variable in the above transcendental equation is the oxygen ion scale height, which is obtained after numerically solving the equation. It is assumed that the ionosphere is isotropic, therefore the $H^+$ scale height will be 16 times larger than the $O^+$ scale height. The correction factor $V$ represents the change from magnetic field line direction to vertical direction (Stankov et al. 2002).

### *2.2 Exponential layer*

The exponential layer is defined as:

$$N_i(h) = N_i(h_m) \exp\left(-\frac{h - h_m}{H_i}\right) , \tag{4}$$

where $N_i(h)$ is the density at height $h$, $H_i$ (positive) is the ion scale height. For the oxygen ion the solution, obtained similarly as for the Sech-squared layer, is given by $\int_{hm}^{hc} N_{O^+}(h)dh = H_{O^+} N_{O^+}(h_m)$, $h_c \gg h_m$. The corresponding equation for the scale height is now:

$$\left(\frac{16V}{(16V-1)} N_m - \frac{1}{(16V-1)H_{O^+}} TEC_t\right) \exp\left(-\frac{h_{tr} - h_m}{H_{O^+}}\right) -$$
$$- \left(\frac{1}{(16V-1)H_{O^+}} TEC_t - \frac{1}{(16V-1)} N_m\right) \exp\left(-\frac{h_{tr} - h_m}{16VH_{O^+}}\right) = 0 .$$

### *2.3 Chapman layer*

The general form of the Chapman layer is defined as

$$N(h) = N(h_m) \exp\left\{c\left[1 - \frac{h - h_m}{H} - \exp\left(-\frac{h - h_m}{H}\right)\right]\right\} , \tag{5}$$

where $h_m$ is the peak density height and $H$ is the scale height, $c$ is the type coefficient. This model has two distinct formulations — the so-called $\alpha$-Chapman layer ($c = 0.5$) and $\beta$-Chapman layer ($c = 1$), depending on assumptions related to the electron recombination theory (Hargreaves 1992). The $\alpha$-Chapman layer assumes that the electrons recombine directly with positive ions and that no negative ions are present, i.e. $X^+ + e \to X$, and the loss rate is then $L = \alpha N^2$, where $\alpha$ is the recombination coefficient. In the $\beta$-Chapman formulation, the assumption is that the electron loss takes place by charge exchange between an ion and neutrals followed by ion-electron recombination with linear loss rate $L = \beta N$, where $\beta$ is the linear recombination coefficient. As height increases, the behaviour changes from $\alpha$ to $\beta$ type at a height where $\beta = \alpha N$.

For the $\alpha$-Chapman layer, the density at a given height is

$$N(h) = N(h_m) \exp\left\{\frac{1}{2}\left[1 - \frac{h - h_m}{H} - \exp\left(-\frac{h - h_m}{H}\right)\right]\right\} . \tag{6}$$

Applying the substitutions

$$x=\left(\frac{h-h_m}{H}\right);\ x_c=\lim_{h\to h_c} x=\frac{h_c-h_m}{H},\ x_m=\lim_{h\to h_m} x=0$$

$$y=\frac{1}{\sqrt{2}}\exp\left(-\frac{x}{2}\right);\ y_c=\lim_{x\to x_c} y=\exp\frac{1}{2}\left(-\frac{h_c-h_m}{2H}\right),\ y_m=\lim_{x\to x_m} y=\frac{1}{\sqrt{2}}$$

the integration of the $\alpha$-Chapman function yields the following formula:

$$\int_{hm}^{hc} N(h)dh = HN_m \int_{xm}^{xc} \exp\left[\tfrac{1}{2}-\tfrac{1}{2}x-\tfrac{1}{2}\exp(-x)\right]dx =$$
$$= HN_m \int_{ym}^{yc} \exp\left[\tfrac{1}{2}+\tfrac{1}{2}\ln(2)+\ln(y)-y^2\right]\left(-\tfrac{2}{y}\right)dy = -2HN_m\sqrt{2e}\int_{ym}^{yc}\exp(-y^2)dy =$$
$$= HN_m\sqrt{2e\pi}\tfrac{2}{\sqrt{\pi}}\int_{yc}^{ym}\exp(-y^2)dy = HN_m\sqrt{2e\pi}\,erf\left(\tfrac{\sqrt{2}}{2}\right)\approx 2.821HN_m$$

because $y_c=0$ for very large upper boundary height, $(h_c-h_m)>20H$. Therefore, the topside electron content is

$$TEC_t = 2.821H_{O^+}N_{O^+}(h_m)+2.821H_{H^+}N_{H^+}(h_m) \tag{7}$$

and the equation for determining the $O^+$ scale height is:

$$\left(\frac{16V}{(16V-1)}N_m-\frac{1}{2.821(16V-1)H_{O^+}}TEC_t\right)\exp\frac{1}{2}\cdot$$
$$\left(1-\frac{h_{tr}-h_m}{H_{O^+}}-\exp\left(-\frac{h_{tr}-h_m}{H_{O^+}}\right)\right)-$$
$$-\left(\frac{1}{2.821(16V-1)H_{O^+}}TEC_t-\frac{1}{(16V-1)}N_m\right)\exp\frac{1}{2}\cdot$$
$$\left(1-\frac{h_{tr}-h_m}{16VH_{O^+}}-\exp\left(-\frac{h_{tr}-h_m}{16VH_{O^+}}\right)\right)=0\,.$$

Similarly, for the $\beta$-Chapman layer the density at a given height is

$$N(h)=N(h_m)\exp\left\{\left[1-\frac{h-h_m}{H}-\exp\left(-\frac{h-h_m}{H}\right)\right]\right\}. \tag{8}$$

The substitutions are now

$$x=\left(\frac{h-h_m}{H}\right);\ x_c=\lim_{h\to h_c} x=\frac{h_c-h_m}{H},\ x_m=\lim_{h\to h_m} x=0$$

$$y=\exp(-x);\ y_c=\lim_{x\to x_c} y=\exp\left(-\frac{h_c-h_m}{H}\right),\ y_m=\lim_{x\to x_m} y=1$$

and the integration is carried out follows:

$$\begin{aligned}
&\int_{hm}^{hc} N(h)dh = HN_m \int_{xm}^{xc} \exp\left[1 - x - \exp(-x)\right] dx = \\
&= HN_m \int_{ym}^{yc} \exp\left[1 + \ln(y) - y\right] \left(-\tfrac{1}{y}\right) dy = -HN_m \int_{ym}^{yc} \exp(1-y)dy = \\
&= HN_m \left\{\exp(1-y)\right\}\Big|_{ym}^{yc} = HN_m(e-1) \approx 1.718 HN_m
\end{aligned}$$

because $y_c = 0$ for very large values of $h_c$, $(h_c - h_m) > 10H$. Then, the topside electron content is

$$TEC_t = 1.718 H_{\mathrm{O}^+} N_{\mathrm{O}^+}(h_m) + 1.718 H_{\mathrm{H}^+} N_{\mathrm{H}^+}(h_m) \tag{9}$$

and the equation for determining the $\mathrm{O}^+$ scale height is:

$$\begin{aligned}
&\left(\frac{16V}{(16V-1)} N_m - \frac{1}{1.718(16V-1)H_{\mathrm{O}^+}} TEC_t\right) \cdot \\
&\cdot \exp\left(1 - \frac{h_{tr} - h_m}{H_{\mathrm{O}^+}} - \exp\left(-\frac{h_{tr} - h_m}{H_{\mathrm{O}^+}}\right)\right) - \\
&- \left(\frac{1}{1.718(16V-1)H_{\mathrm{O}^+}} TEC_t - \frac{1}{(16V-1)} N_m\right) \cdot \\
&\cdot \exp\left(1 - \frac{h_{tr} - h_m}{16V H_{\mathrm{O}^+}} - \exp\left(-\frac{h_{tr} - h_m}{16V H_{\mathrm{O}^+}}\right)\right) = 0\,.
\end{aligned}$$

Note that, the height $h_c$ is practically infinity in the case of GPS measurements since the electron density above the mean height of the plasmapause contributes by a negligible quantity to the integrated electron content.

The topside profiles produced by the above models tend to asymptotically approach the exponential profile at great altitudes ($h \gg h_m$F2). The approaching 'speed' is different for different layers and the $\alpha$-Chapman is obviously the slowest (Fig. 1).

The parabolic layer is defined as $N(h) = N(h_m)\left\{1 - ((h-h_m)/2H)^2\right\}$, where typical values of $H$ for the F2 region lie in the range 25 to 50 km (Dieminger et al. 1995). This layer is suitable for modelling the profile near $h_m$F2 and is useful when extracting information from the satellite data. It is also good for constructing composite ionospheric models. However, as a reconstruction tool, used separately, it is certainly not appropriate.

The upper transition level, $h_{tr}$, is determined from an empirical model (Kutiev et al. 1994) based on satellite and rocket measurements. In this model, the UTL values are approximated by a multi-variable polynomial, supplying the required values according to the input geophysical parameters like solar activity, latitude, local time, etc.

## 3. Satellite in-situ measurements for comparison purposes

Density profiles, obtained by the analytical models described in the previous section, will be compared with independent measurements. Relatively good altitude profiles of ion densities can be obtained from the Atmosphere Explorer – C (AE-C) satellite in-situ measurements.

The satellite was launched on 13/12/1973 in an elliptical orbit (inclination 68.1°) collecting a large data base of ionospheric and thermospheric densities, temperatures, winds, and emissions within the altitude range of 130–4300 km. After the first eight months, the mode was changed and the spacecraft was kept in a circular orbit for the rest of its lifetime (re-entry date 12/12/1978): from March 1975 to December 1976 at a height of about 300 km, and from December 1976 to December 1978 at a height of about 400 km.

The $O^+$ and $H^+$ ion density data used here were obtained during the first 16 months of the AE-C mission, from 16/12/1973 to 21/3/1975 when the solar activity was low, F10.7 $\approx$ 85 W/m$^2$/Hz. The data were measured by the Bennett and Magnetic ion-mass spectrometers. Three seasons are considered – winter, equinox, and summer, defined as 91 day periods centred on the 356, 81 and 264, 173 day of the year respectively. Day-time and night-time conditions are investigated using data from variable local time ranges (windows) depending on the season. Larger day-time windows (0800–1700) are used for summer values and larger night-time windows (1900–0500) are selected in case of the winter data. Middle geomagnetic latitudes ($20-50$°N) and the Northern hemisphere will be only presented. Averaged profiles are presented in Fig. 2 and Fig. 3 for $O^+$ and $H^+$ densities.

In order to compare the analytical models with the actual data, some basic density profile characteristics should be found from the averaged data. Particularly important for the unique determination of a profile are the F2-layer density maximum and height and also the $O^+-H^+$ transition level (UTL). These characteristics can be determined as described below and indicated in Fig. 2.

The first important characteristics to extract are the density peaks and their heights for both the $O^+$ and electron density profiles. If quality data are available the extraction is straightforward. If not, the parabolic layer can be used to fit the data near the F2-layer density maximum.

The next important parameter to be obtained from the data is the $O^+$ ion scale height, $H_{O^+}$. The scale height is defined as the vertical distance in which the concentration decreases by a factor of $e$ ($e \approx 2.718286$). This definition allows the determination of the scale height from the average satellite measurements. The scale height varies with height but at this stage of developing the method it is assumed to be constant. Because of this assumption, it is important to deduce the scale height from the area immediately above $H_m$F2 which contributes most to the TEC value. The curvature of the $O^+$ density profile does not allow to use for the determination of as lower boundary $H_m$F2 so it starts at an altitude $h_m\text{F2} + h_\varepsilon$, where $h_\varepsilon$ is approximately the half thickness of the fitting parabola. There are plenty of measurement data in this region and the scattering is generally small.

The propagation factor M(3000)F2 is obtained by fitting the satellite data to

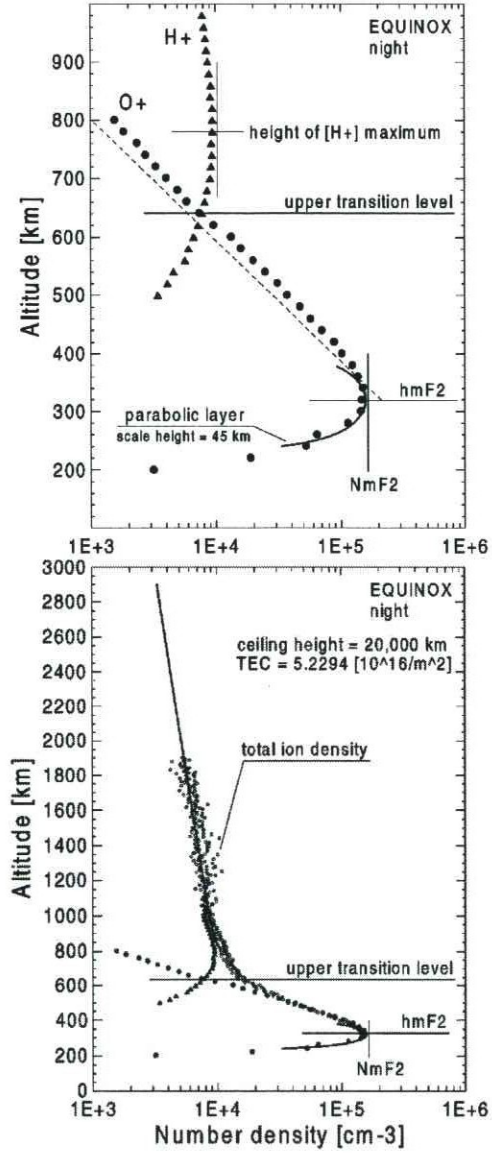


Fig. 2. Basic profile characteristics deduced from satellite data: HmF2, NmF2, scale height, and upper transition level (top panel) and TEC value (bottom panel)

Epstein layer functions. The E-layer critical frequency, $f_o$E, is zero during night and the day-time values are some plausible values characteristic for the season and local time at the corresponding latitude. For the purpose of this study the precise determination is not important as the bottom-side electron profile is fitted with the same Epstein layer that is used in the reconstruction technique.

The $O^+-H^+$ transition height is relatively easy to determine during night when $O^+$ scale height is small. During daytime the data scattering is larger because of the strong latitude dependence. In such cases, a power approximation of the oxygen ion profile near the UTL region helps usually.

The last characteristic to determine is the total electron content value. This is done by integrating the satellite-data-based electron density profile. In all cases the electron density profile has to be extrapolated to some large altitude. For this purpose, the hydrogen ion scale height is first deduced from the available data above the $H^+$ density peak and then an exponential layer is employed to simulate the distribution up to the upper boundary height of 20000 km. The profile-based TEC value is calculated using quadratures.

Averaged altitude profiles are shown in Fig. 2 and Fig. 3 where the symbols represent ion density averaged over 20 km in altitude. The total ion density is indicated by open circles for averages over 5 km. In case of Fig. 2, TEC is approximately $5.2294 \times 10^{16}$ m$^{-2}$.

## 4. Results and discussions

There are two types of tests that have been carried out to evaluate the efficiency of the exponential, sech-squared and Chapman (both $\alpha$ and $\beta$ type) layers as reconstruction tools. First, the vertical distribution is obtained with fixed $O^+$ scale height, peak density and peak height. It aims at comparing the overall simulation ability of the profiler. Second, given a TEC value, fixed for given season and local time, the reconstruction is performed with each profiler and the results are analysed and compared. This task aims at determining the most suitable profiler from reconstruction point of view.

### *4.1 Height distributions obtained with a same scale height*

The vertical density distribution produced with equal scale height (100 km) has already been presented in Fig. 1 for the four profiles.

In order to determine which model gives better results for the $O^+$ density profile, all input parameters have been extracted from the averaged $O^+$ profiles obtained from AE-C data as described in the previous section. The required parameters are: the $O^+$ peak height ($h_m$), the peak density ($N_{O^+}(h_m)$), and the scale height ($H_{O^+}$). The retrieved parameters are listed in Table I for the three seasons of interest (winter, equinox, summer), and for day-time and night-time conditions.

The vertical ion density distributions are given in Fig. 3. Same scale height was used for each season and local time conditions. Only the sech-squared and exponential layers are plotted in the figure, as the Chapman profiles indicate the same features (as compared to the above layers) already presented in Fig. 1. It is obvious that the different profilers produce quite different altitude distributions in the F region and therefore they will produce significant differences in the corresponding values of the total electron content. The largest contribution for the TEC value

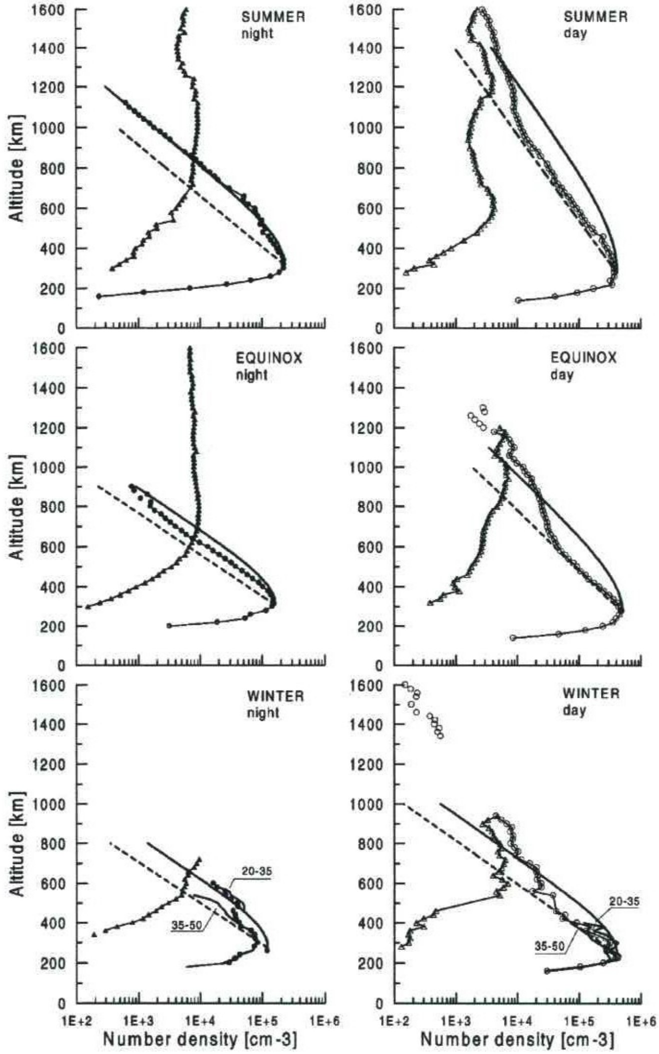


Fig. 3. Sech-squared (solid line) and exponential (dashed line) $O^+$ profiles compared with AE-C measurements of $O^+$ (circles) and $H^+$ (triangles) densities from the 20 − 50° N geomagnetic latitude range. For winter-time the averaged profiles from 20 − 35° N and 35 − 50° N are also given

comes from the region near the peak height and it is most important to compare the efficiency mainly in this region.

It seems that the sech-squared model is more suitable for describing the night-time behaviour, while the day-time behaviour is better represented by the exponential and $\beta$-Chapman. The latter ensures better simulation in the region near the peak. The sech-squared and $\alpha$-Chapman layers generally overestimate the day-time values. If the profiles are forced to pass through a given point, i.e. given density at a given height, then the shape changes as it will be seen below.

**Table I.** The basic $O^+$ density profile characteristics obtained from AE-C data

| AE-C data | Winter | Equinox | Summer |
|---|---|---|---|
| F10.7 $W/m^2/Hz$ | 80.18 | 84.48 | 90.20 |
| dipole latitude, deg | 20 – 50°N | 20 – 50°N | 20 – 50°N |
| | Day-time | | |
| local time, hour | 0800–1600 | 0800–1630 | 0800–1700 |
| $N_{O^+}(h_m)$, $cm^{-3}$ | 4.03 × 105 | 4.96 × 105 | 4.10 × 105 |
| $h_m$, km | 245 | 275 | 280 |
| $H_{O^+}$, km | 95 | 130 | 185 |
| | Night-time | | |
| local time, hour | 1900–0500 | 2000–0430 | 2000–0400 |
| $N_{O^+}(h_m)$, $cm^{-3}$ | 1.20 × 105 1.58 × 105 | 3.00 × 105 | |
| $h_m$, km | 275 | 320 | 320 |
| $H_{O^+}$, km | 90 | 90 | 110 |

### *4.2 Reconstruction using a same TEC value*

The vertical ion and electron density distribution produced with the same scale height (100 km) has been shown in Fig. 1 for the four profiles. The largest values at all altitudes has been obtained with the $\alpha$-Chapman profiler and the smallest — with the exponential layer. In order to investigate the reconstruction quality of each profiler, it will be necessary to find out how the reconstructed profiles will look like in case of fixed TEC and UTL values.

The results in the previous chapter suggest that in order to preserve the same TEC value, the calculated $O^+$ scale height using the reconstruction method should be the largest if the exponential layer is used, and inversely, it should be smallest if the $\alpha$-Chapman layer is used. In order to prove that, the method was tested for a fixed upper transition level and fixed TEC value of $15.0 \times 10^{16}$ $m^{-2}$. The results are presented in Fig. 4 for UTL = 1500 km (top panel) and 1000 km (bottom panel).

As expected, the scale height, obtained via the exponential layer, is the highest: $H_{O^+}$ = 244 km for UTL = 1500 km and $H_{O^+}$ = 194 km for UTL = 1000 km, which yields much steeper electron density profile. Lowest scale height values are obtained with $\alpha$-Chapman: 94 and 78 km respectively. However, because of the profilers definitions and the different way of simulating the density distribution near the maximum height $h_m$, the reconstructed Chapman-like distributions both lie in-between the 'boundary' profiles obtained with the exponential and sech-squared profilers. Also, because of the differences at $h_m$, all reconstructed profiles intersect at a given altitude somewhere below UTL.

As a result of the differences in the calculated oxygen ion scale heights, significant differences are observed in the hydrogen ion density profiles ű both in scale height

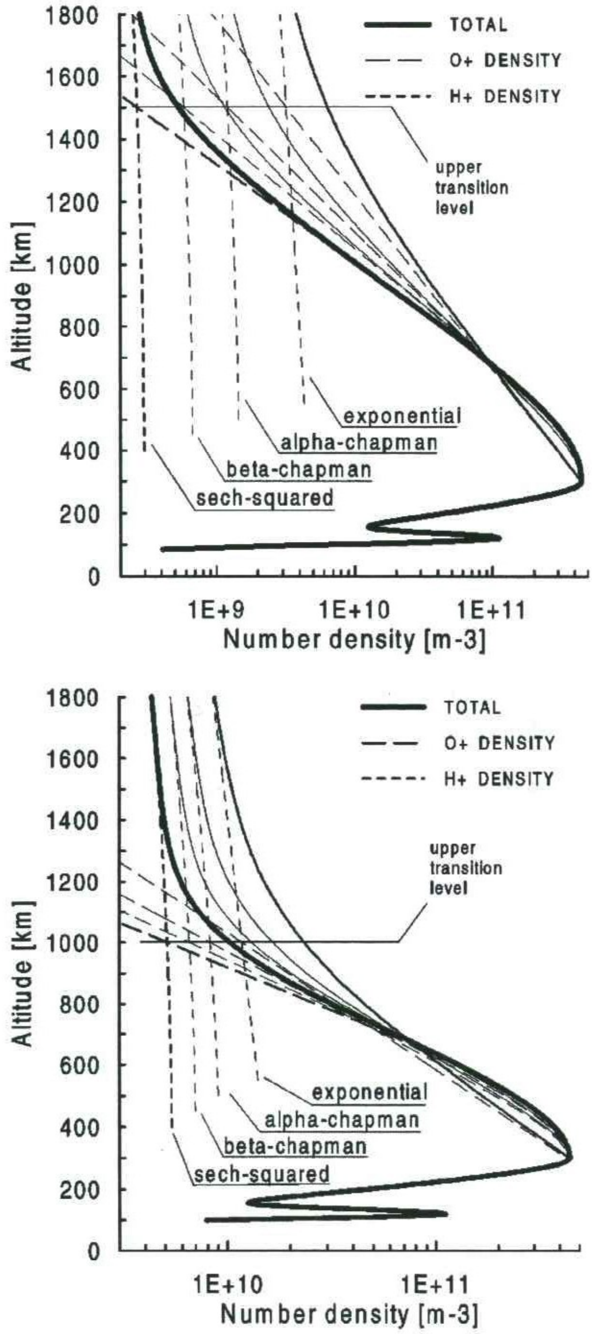


Fig. 4. Comparison between the vertical electron density profiles obtained with equal $TEC = 15.0 \times 10^{16}$ [m−2] for UTL = 1500 km (top) and UTL = 1000 km (bottom). The calculated $O^+$ scale heights (for UTL = 1500/1000): exponential = 244/194 km, sech-squared = 136/120 km, $\alpha$-Chapman = 94/78 km, $\beta$-Chapman = 157/135 km

and in absolute ion density values. This differences are increasing if the upper transition level is increased (Fig. 4), which once again underline the importance of correct UTL values. The influence of the UTL on the scale-height calculations has been already analysed for the sech-squared profiler (Stankov et al. 2002).

The next step in the evaluation procedure is to deduce the density profiles using the reconstruction technique and evaluate them against averaged data profiles. The results for equinox night and day-time conditions will be discussed in more detail because the available data are more reliable.

*Night-time conditions*

These values and the other profile characteristics (Table I), are used to reconstruct the profiles for the four models investigated. The results are presented in the top four panels of Fig. 5. In order to enable the better analysis the quality of the reconstruction, the relative errors are presented in the same figure for the oxygen and electron profiles. At first glance, it is obvious that no single profile is significantly better than the other if the reproduction of the ion density distribution is considered. The $O^+$ profile is best modelled by the $\alpha$-Chapman layer at lower altitudes and equally good by the $\beta$-Chapman and sech-squared profilers near the upper transition layer. The region above UTL (640 km) is not so important in case of the $O^+$ profile. The situation is slightly different for the electron density profile. Starting from $H_m$F2, the sech-squared and Chapman profilers represent very good approximation up to about 400 km. From this altitude, up to the altitude of the 'profile crossing' point (560 km), the Exponential layer is much better than the others. For the region above, the sech-squared layer and $\beta$-Chapman are the better choice. Overall, for night-time electron density distribution at equinox it is best if the Chapman profiles are used — the $\alpha$ type at lower altitudes and $\beta$ type for the region above the intersecting point. The calculated TEC is $5.2294 \times 10^{16}$ m$^{-2}$.

*Day-time conditions*

The averaged $O^+$ profiles show rather large variability in the scale height. It is mainly due to the strong altitude variations in the ion temperatures during day-time. Also, the density distribution, and consequently the $O^+ - H^+$ TL, vary strongly in latitudinal direction, so the averaged profiles reflect these facts. In order to decrease the influence of the latitude dependence, data have been determined from a smaller latitude band (27.5° – 42.5°N ). The density scattering is still high and, the $O^+$ and $H^+$ profiles have been calculated. The $O^+$ profile has been fitted (power approximation, standard deviation = 0.045) in the region from 640 to UTL (1330 km). Above the $H^+$ profile maximum (~1020 km), the $H^+$ density was again extrapolated using an exponential layer with scale height approximately 16 times larger than the $O^+$ scale height near this altitude (1020 km). The electron density profile is calculated from the fitted $O^+$ and $H^+$ densities. All reconstructed profiles are given in the top four panels of Fig. 6, and the relative errors using the fitted profiles are given at the bottom of the figure. During day-time, the exponential layer is undoubtedly the best choice. It provides better overall results for the individual ion density profiles as well as for the total ion/electron density. The estimated day-time TEC value is much higher, $12.3820 \times 10^{16}$ m$^{-2}$.

For summer conditions, the best options are the sech-squared layer for night-time and the exponential profiler for day-time conditions. During winter the data are scarce and highly scattered and it is difficult to draw decisive conclusions. However, the winter values at least prove the necessity of a more detailed look on the reconstructed patterns in latitude direction. The results in Fig. 3 suggest that at lower latitudes the sech-squared profiler provides better results at night and at higher latitudes — the Chapman profilers (Fig. 5). The exponential layer is again the best option for the day-time conditions (Fig. 6).

Uncertainties in the evaluation procedure have been introduced via possible incorrect determination of TEC and also through the larger values of $H^+$ density below the $H^+$ peak because of the assumption for equal heights of the $O^+$ and $H^+$ density maxima. Additional studies are required for high-solar activity conditions.

## 5. Conclusions

The reconstruction of the electron density height profile from TEC measurements, using UTL and ionosonde data, is a technique utilising different types of measurements to solve a long-lasting ionospheric physics problem. The efficiency of the most popular analytical ionospheric models (Chapman, Sech-squared, and exponential) has been evaluated as they are important ingredients in the method of reconstructing the topside ion/electron density profiles. Particular care was taken of the calculation of the scale height in the upper ionosphere, from the F2-layer density peak height up to the $O^+$–$H^+$ transition level. The $O^+$ ion scale height is the most important unknown parameter and it should be determined in a most precise way.

The main conclusions are the following:

— The Sech-squared, exponential, and Chapman profilers can all be successfully used in the reconstruction technique. New formulae have been derived.

— No single profiler can sufficiently well represent all spatial and temporal variations. The tests with different simple layers suggest that each of them simulate the topside ionosphere in a different manner in the upper ionosphere region, leading to significant differences that affect the TEC calculations. Differences are observed even in latitudinal direction suggesting that a composite profiler should be considered.

— The density profiles produced by the Sech-squared and Chapman models tend to asymptotically approach the exponential layer distribution at great altitudes.

— For day-time conditions, the exponential layer is more suitable than the other profilers for modelling/reconstruction purposes.

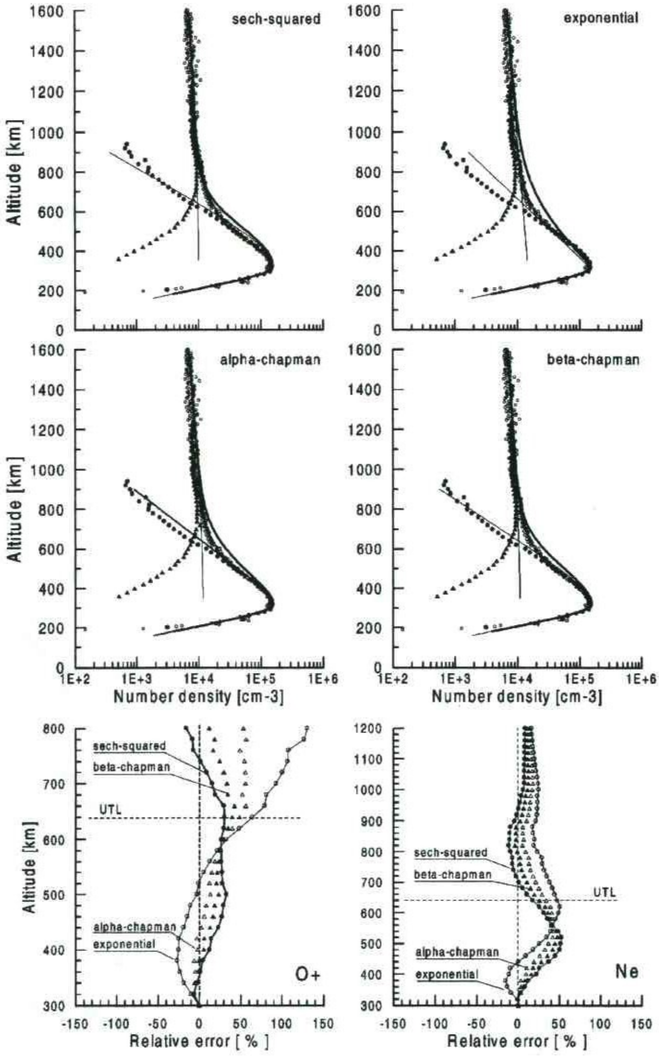


Fig. 5. Profile reconstruction (night-time) using Sech-squared, xponential, and Chapman profilers; comparison with AE-C data, $TEC = 5.2294 \times 10^{16}$ [$\mathrm{m}^{-2}$], UTL = 640 km

— For night-time conditions, the Sech-squared and Chapman models guarantee better reconstruction results.

— The parabolic layer is not suitable for reconstruction but is a very helpful tool for simulation and determination of profile characteristics near the density maxima.

— There is a pronounced need for a high-quality ionosphere-plasmasphere temperature model which will help to determine the topside scale heights much more precisely.

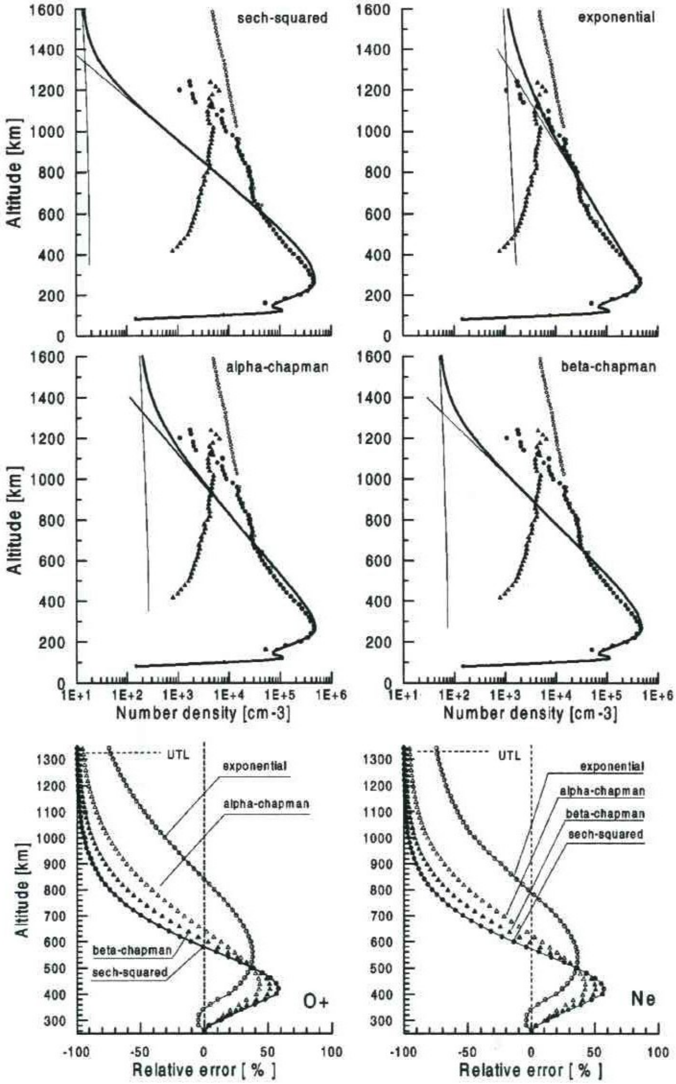


Fig. 6. Profile reconstruction (day-time) using Sech-squared, exponential, and Chapman profilers; comparison with ion profiles fitted to the AE-C data, $TEC = 12.3820 \times 10^{16}$ [$m^{-2}$], UTL = 1330 km

— The upper transition level plays a key role in the reconstruction procedure. Inaccurate values of the transition level, especially during day-time, may cause quite large errors in the topside distribution due to the high sensitivity of the $H^+$ profile.

The reconstruction technique is a powerful research instrument and can be further improved when more measurements are available from various sources.

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