Acta Geodaetica et Geophysica Hung. (2002), Vol. 37(2-3), pp.171-181.

ELECTRON DENSITY PROFILES DEDUCED FROM GPS TEC, O^{+}- H^{+} TRANSITION HEIGHT AND IONOSONDE DATA

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[Manuscript received December 17, 2001]

A new formula of the electron density profiles above the peak height is introduced. The formula is based on the Epstein layer and depends on the O^+ and H^+ scale heights and the O^+-H^+ transition level. Both scale heights have a ratio 1:16 reduced by a factor representing the change from magnetic field line direction to vertical direction. The bottom-side part of TEC (calculated by using foF2, $M_{3000}F2$ and foE measurements) is subtracted from the GPS-derived TEC at the same location. The topside TEC, together with the empirically obtained O^+-H^+ transition level, are then used to deduce the unknown scale heights. The method is demonstrated on actual data covering low and high solar activity conditions.

Keywords: GPS, TEC, electron profile reconstruction, transition level

1. Introduction

Given the electron density profile (i.e. the altitude distribution of the electron density), it is relatively easy to calculate the corresponding total electron content (TEC) using quadrature formulae. The purpose of this paper is to present a method for solving the inverse problem - deducing the electron density profile from the total electron content.

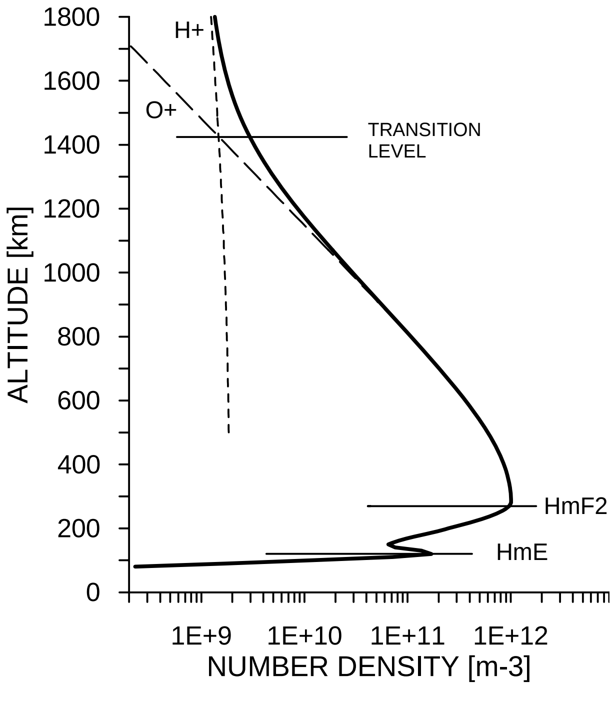


Fig.1 A two-slope electron density profile (solid line): topside part obtained after summing up the O+ (long dashes) and H+ (short dashes) ion densities.

The GPS-measured TEC delivers the basic *quantitative* information about the sought electron profile. Additional information about the *shape* of the electron profile is also required, e.g. maximum density and height of the E and F layers, the curvature of the topside profile near the O^+ - H^+ (upper) transition level (TL), etc. (**Fig.1**).

While the ionosonde measurements are sufficient for determination of the bottom-side parameters of the profile, they do not provide information about the topside part of the profile. Even if we know the F layer peak density and height, we cannot determine the topside electron distribution because the plasma scale height is unknown. The upper transition level (if available) is the reference point we need to calculate the plasma scale height. Then, assuming an adequate topside density distribution law we can tie the profile to the F layer peak height and the O^+ - H^+ transition height. We still have to observe the fulfillment of the most important quantitative requirement - the calculated TEC (sum total of the integrated bottom-side and top-side electron density) should equal the measured TEC.

2. Method

The total electron content is split into a bottom-side, TEC_b , and a topside, TEC_t , contents:

$$TEC = TEC_b + TEC_t = \int_0^{h_m} N_e(h)\,dh + \int_{h_m}^{\infty} N_e(h)\,dh$$

where $N_e(h)$ is the electron density at height h and h_m is the F2 peak height.

The *bottom-side* electron profile and corresponding *bottom-side* electron content are reliably calculated from foF2, $M_{3000}F2$ and foE using established methods and models (Bradley and Dudeney 1973, Dudeney 1978, Dudeney 1983, Bilitza et al. 1993, Di Giovanni and Radicella 1990, Radicella and Leitinger 2001).

This study is focused on the determination of the *topside* electron profile, presented as a sum of its major constituent oxygen and hydrogen ion density profiles. Further, the individual (oxygen and hydrogen) ion density distributions are approximated by the hyperbolic secant function in the following manner:

$$N_i(h) \quad = \quad N_i(h_m)\, sech^2\left(\frac{h-h_m}{2\,H_i}\right) \tag{1}$$

where $N_i(h)$ is the ion (O^+ or H^+) density at height h, H_i is the ion scale height, and $sech(h) = 1/\cosh(h)$, $\cosh(h) = 0.5\left(\exp(h) + \exp(-h)\right)$. Therefore, the following 'reconstruction' formula is proposed for calculation of the topside electron density profile:

$$N_e(h) = N_{O+}(h_m)\, sech^2\left(\frac{h-h_m}{2\,H_{O+}}\right) + N_{H+}(h_m)\, sech^2\left(\frac{h-h_m}{32\,H_{O+}}\right) \quad , \quad h > h_m \tag{2}$$

where H_{O+} is the O^+ scale height. The first term on the right represents the O^+ vertical distribution, while the second term represents that of H^+. If we consider an isotropic ionosphere and plasmasphere (constant electron temperature), then the scale heights of O^+ and H^+ along the magnetic field lines will have a ratio 1:16. Here we neglect the fact that H^+ has a maximum above the transition height and assume that H^+ decreases exponentially from the level of h_m . This is true at altitudes well above the transition height. $N_{O+}(h_m)$ and the virtual quantity $N_{H+}(h_m)$ are the respective densities at the height of the F2 peak. To obtain the profile on the vertical direction, z , we use the simple conversion dz = sinI ds , where ds is the differential element along the field lines, I is the inclination. If we ignore the displacement of the geographic and magnetic poles, then dz = sin[arctg(2tgφ)]ds , where φ is the latitude (Chapman 1963). Denoting V=sin[arctg(2tgφ)], equation (**2**) takes the form

$$N_e(h) = N_{O+}(h_m)\, sech^2\left(\frac{h-h_m}{2\,H_{O+}}\right) + N_{H+}(h_m)\, sech^2\left(\frac{h-h_m}{32\,VH_{O+}}\right) \quad , \quad h > h_m \tag{3}$$

There are three unknown variables in the proposed formula - the oxygen and hydrogen ion densities at the peak height, i.e. $N_{O+}(h_m)$ and $N_{H+}(h_m)$, and the oxygen ion scale height H_{O+} . These unknowns are determined in the following way.

After integrating $N_e(h)$ from h_m to infinity, the above 'reconstruction' formula (**3**) becomes

$$\int_{h_m}^{\infty} N_e(h)dh = \int_{h_m}^{\infty} N_{O+}(h_m) sech^2\left(\frac{h-h_m}{2\,H_{O+}}\right)dh + \int_{h_m}^{\infty} N_{H+}(h_m) sech^2\left(\frac{h-h_m}{32\,V\,H_{O+}}\right)dh \tag{4}$$

hence

$$TEC_t \quad = \quad 2\,H_{O+}\;N_{O+}(h_m) \quad + \quad 32\,V\,H_{O+}\;N_{H+}(h_m) \tag{5}$$

Denoting, $N_{O+}(h_m) + N_{H+}(h_m) = N_m F2$, we get from equation (**5**) the following expressions for the peak densities:

$$N_{O+}(h_m) = \frac{16V}{(16V-1)} N_m - \frac{1}{2(16V-1)H_{O+}} TEC_t \qquad (6)$$

$$N_{H+}(h_m) = \frac{1}{2(16V-1)H_{O+}} TEC_t - \frac{1}{16V-1} N_m \qquad (7)$$

Considering the assumed type of the topside profile (**1**), and expressions (**6**) and (**7**), the following equation is constructed, denoting the fact that the hydrogen and oxygen ion densities are equal at the O^+-H^+ transition level:

$$\left(\frac{16V}{(16V-1)} N_m - \frac{1}{2(16V-1)H_{O+}} TEC_t\right) sech^2\left(\frac{h_{tr}-h_m}{2\,H_{O+}}\right) -$$

$$-\left(\frac{1}{2(16V-1)H_{O+}} TEC_t - \frac{1}{16V-1} N_m\right) sech^2\left(\frac{h_{tr}-h_m}{32\,V\,H_{O+}}\right) = 0$$

The upper transition level, h_{tr} , is determined from a model (Kutiev et al. 1994). The only unknown variable in the above transcendental equation is the oxygen ion scale height, which is obtained after numerically solving the equation.

3. Data

Three types of data are required for implementing the method - TEC, vertical incidence sounding (ionosonde) and O^+-H^+ transition height data.

3.1 Total Electron Content data

After determining the total electron content along a number of ray path's by using a special calibration technique for the ionospheric delay of GPS signals (Sardon et al. 1994), the slant TEC is mapped to the vertical by using a single layer approximation for the ionosphere at h_{sp}=400 km height. Using the GPS ground stations of the European IGS network, about 60-100 TEC data points are available for reconstructing TEC maps over the area 20°W $\le \lambda \le$ 40°E; 32.5°N $\le \varphi \le$ 70°N. To ensure a high reliability of the TEC maps also in case of only a few measurements or at greater distances from measuring points, the measured data are combined with the empirical TEC model NTCM2 (Jakowski 1996).

For each grid point value (spacing is 2.5°/5° in latitude/longitude) a weighting process between nearest measured values and model values is carried out. The achieved accuracy for TEC is in the order of 2-3 $\times 10^{16} m^{-2}$ (Jakowski et al. 1996) . To derive TEC over the ionosonde stations considered in this study, a linear interpolation algorithm within the corresponding grid pixel is applied.

3.2 Ionosonde data

The information about the bottom-side part of the profile and the electron peak density and peak height is taken from ionosonde measurements; required ionosonde parameters are the F2–layer critical frequency (foF2), the propagation factor (M_{3000}F2), and the E–layer critical frequency (foE). The F2-layer peak height is estimated using the expression (Dudeney 1983):

$$hmF2 = -176 + 1470 \frac{M_{3000}F2 \left\{(0.0196\, M_{3000}F2^2 + 1)/(1.296\, M_{3000}F2^2 - 1)\right\}^{1/2}}{M_{3000}F2 - 0.012 + 0.253/(foF2/foE - 1.215)}$$

The bottom-side thickness, B_{bot}, is calculated by (Di Giovanni and Radicella 1990) $B_{bot} = 0.385\, N_m F2\, (dN/dh)^{-1}_{\max}$, where $(dN/dh)_{\max}$ is the value of the gradient of $N_e(h)$ at the base of the F2 layer, and it is determined by the following formula:

$$(dN/dh)_{\max}[10^9 m^{-3} km^{-1}] = \exp\left(-3.467 + 0.857 \ln(foF2[MHz])^2 + 2.02 \ln(M_{3000}F2)\right)$$

When F2 and E layers are both present in the ionograms, the bottom-side profile is constructed as a sum of two identical Epstein layers (Rawer 1988):

$$N(h) = 4\, N_m \exp\left((h-h_m)/B_{bot}\right) \left(1+\exp\left((h-h_m)/B_{bot}\right)\right)^{-2},$$

where N_m and h_m are the (F2- or E-) layer's peak density and peak height respectively. The electron density distribution at D region heights is not modelled in detail.

3.3 Upper transition level

The relative abundance of hydrogen ions is a significant factor affecting the topside electron density profile, hence the O^+-H^+ transition level can be successfully utilized as a reference point. This transition level is particularly useful because: a) it is always above the F layer peak height; b) it can be determined independently (from satellite measurements). The level, h_{tr} , is determined from a model (Kutiev et al. 1994), based on satellite in-situ measurements of the individual O^+ and H^+ ion densities. In this model, the transition level is approximated by a multi-variable polynomial, providing convenience when referencing the level with respect to solar activity, season, local time, longitude and latitude:

$$P(C,N;X) = \sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}\sum_{i_3=1}^{n_3}\sum_{i_4=1}^{n_4}\sum_{i_5=1}^{n_5} C(i_1,i_2,\ldots,i_5).\, g_1(i_1,x_1).\, g_2(i_2,x_2)\ldots g_5(i_5,x_5)$$

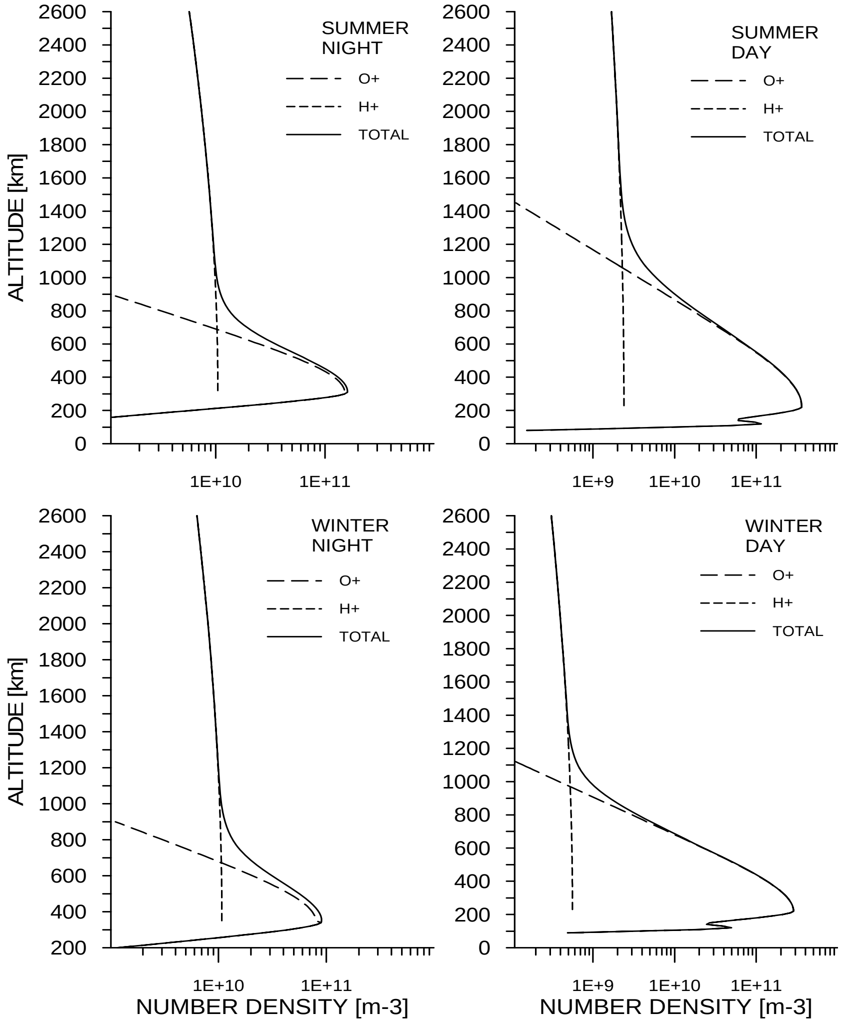
where C={ $C(i_1,i_2,\ldots,i_5)$, i_m=1,..., n_m} are the coefficients and $\left\{g_m(i_m,x_m)\right\}_{i_m=1}^{n_m}$ is a system of linearly independent functions on the domain of the m^{-th} parameter x_m , e.g. algebraic basis($1, x, x^2, \ldots, x^{n_m}$), trigonometric basis ($1, \sin x, \cos x, \ldots, \sin n_m x, \cos n_m x$), etc. The method of least-squares fit is applied for determining the coefficients.

4. Results and discussion

The electron density profile reconstruction is demonstrated below using the required input: GPS-measured total electron content, empirically obtained upper transition level, F2 and E layers' critical frequencies, and propagation factor. The method was tested for various geophysical conditions - low (LSA) and high (HSA) solar activity, winter and summer, night-time (00:00UT) and day-time (12:00UT) conditions. Provided are results for station Juliusruh (13.38E, 54.63N). The scale-height corrector V is set to 0.94.

4.1 Low Solar Activity

The results for LSA, F10.7 ≈ 70 [W/m^2/Hz], are given in **Fig.2**. At summer (top panels), the total electron content increases from 6.5 TECU (TECU = 10^{16}[m^{-2}]) at night to 12.2 TECU at noon.



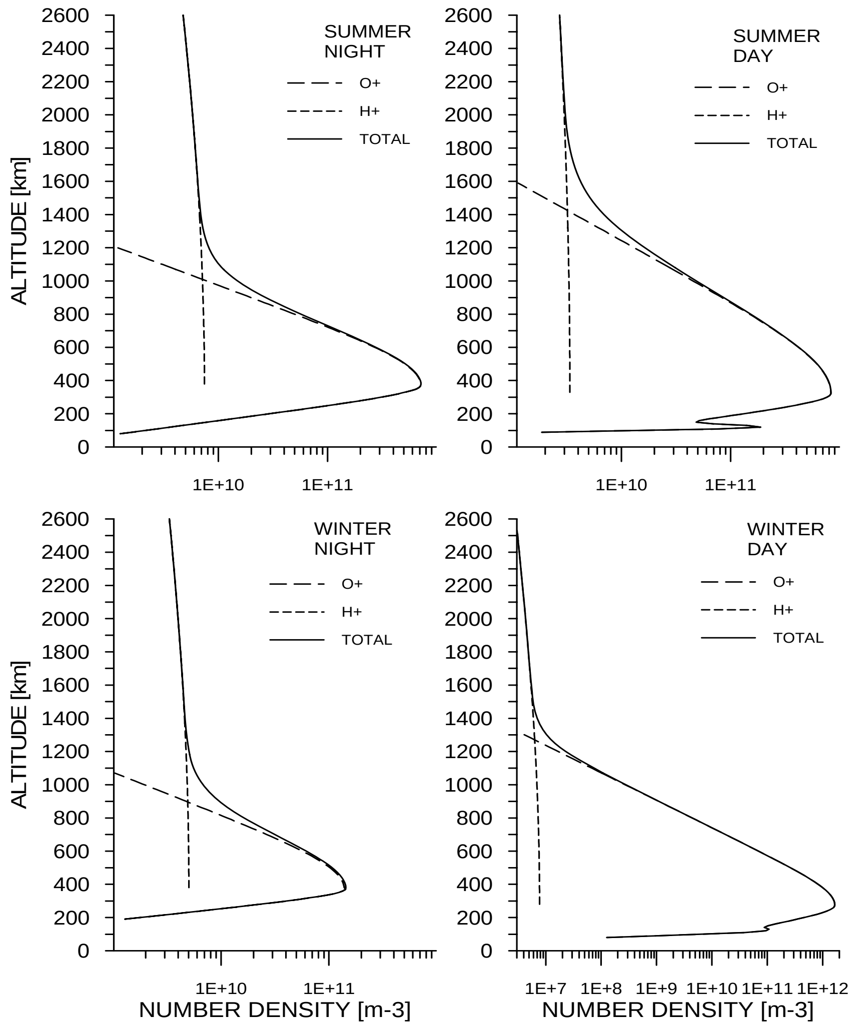
Date	*UT*	*F10.7*	*foF2*	*foE*	$M_{3000}F2$	*TL*	*TEC*	H_{O+}
21/06/1996	12:00	72.60	5.40	3.05	3.25	1059.9	$12.2\text{x}10^{16}$	129.9
21/06/1996	00:00	72.60	3.60	0.00	3.00	688.1	$6.5\text{x}10^{16}$	93.1
21/12/1995	12:00	67.80	4.80	2.00	3.55	969.3	$6.9\text{x}10^{16}$	97.5
21/12/1995	00:00	67.80	2.70	0.00	2.80	672.3	$5.2\text{x}10^{16}$	98.1

Fig.2 Reconstructed electron density profile (solid line) over station Juliusruh (13.38E, 54.63N) for summer (top) and winter (bottom) solstices during low solar activity.

During winter (bottom panels), the total electron contents are much lower - 5.2 TECU at midnight (bottom left) to 6.9 TECU at noon (bottom right). The O^+-H^+ transition level also rises: from 670-690 km at night up to 970-1060 km at noon. Daytime (right panels), the E layer is clearly observed, with its peak density, N_mE, reaching 1.1×10^{11}[m^{-3}] during summer and 5.1×10^{10}[m^{-3}] during winter. During night-time, the E layer is not present and the electron profile falls steeply to zero at around 90 km height.

4.2 High Solar Activity

The results for HSA, F10.7 ≈ 190 [$W/m^2/Hz$], are provided in **Fig.3**. At summer (top panels), the total electron content increases slightly from 22.9 TECU at midnight to 34.9 TECU at noon. At winter (bottom panels) huge differences are observed, the total electron contents jumps from 5.6 TECU at midnight (bottom left) to 34.0 TECU at noon (bottom right). The O^+-H^+ transition level also rises: from about 900/1000 km at night up to 1300/1400 km at noon.



Date	*UT*	*F10.7*	*foF2*	*foE*	$M_{3000}F2$	*TL*	*TEC*	H_{O+}
21/06/2000	12:00	194.20	8.20	3.90	2.65	1425.1	34.9×10^{16}	158.6
21/06/2000	00:00	194.20	7.60	0.00	2.65	1010.0	22.9×10^{16}	106.2
21/12/2000	12:00	188.20	11.60	2.75	3.20	1328.1	34.0×10^{16}	71.8
21/12/2000	00:00	188.20	3.40	0.00	2.65	896.0	5.6×10^{16}	110.6

Fig.3 Reconstructed electron density profile (solid line) over station Juliusruh (13.38E, 54.63N) for summer (top) and winter (bottom) solstices during high solar activity.

The observed (stronger than expected) decrease of the topside electron density profile at winter noon is explained with the unusually large F2 peak density and (probably) overestimated transition level.

4.3 Diurnal behaviour reconstruction

Another example of density reconstruction is provided below in order to better examine the technique over an extended (24-hour) period of time. The example covers the diurnal behaviour of the vertical electron density distribution during summer solstice at high solar activity for station JR055; results are plotted in **Fig.4**.

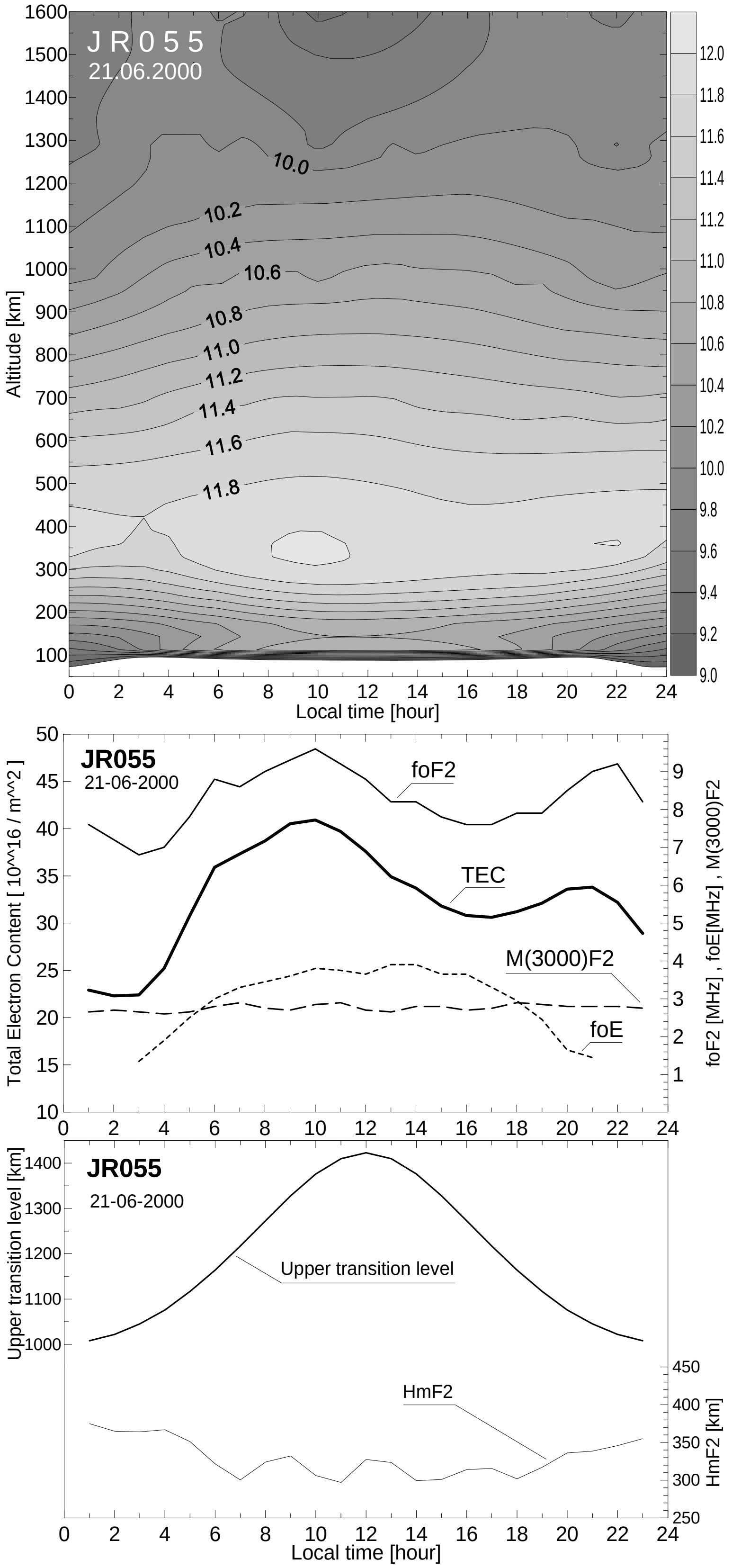


Fig.4 Diurnal behaviour reconstruction: reconstructed vertical electron density (log scale, m^{-3}) distribution (top panel), GPS TEC and ionosonde measurements (middle panel), upper transition level model (bottom panel).

The input values of the GPS-derived TEC, together with the vertical sounding measurements of foF2, M(3000)F2 and foE, are given in the middle panel of **Fig.4**. The TEC and foF2 show strongly correlated diurnal behaviour. Both quantities increase sharply in the early morning, reach their absolute maximum just before noon, and then start gradually decreasing. Relatively high values are maintained throughout the afternoon, followed even by a 10% increase in the early evening. After that, both TEC and foF2 fall rapidly to their corresponding absolute minima observed at 0200-0300LT.

In the bottom panel of **Fig.4** the empirically-modelled heights of F2-peak-density and O^+-H^+ ion transition are also provided. The transition level, starting from 1000 km at midnight, increases up to slightly above 1400 km at noon and then decreases in a symmetrical fashion during the second half of the day. On the other hand, HmF2 has highest values at midnight (around 400 km) and lowest values during day (varying between 300 and 330 km).

The reconstructed electron density distribution is plotted in the top panel of **Fig.4**. Note the detailed vertical distribution above the HmF2. It is easy to detect the changes in the calculated topside scale height and the resulting density distribution as they develop during this particular 24-hour period.

4.4 Effect of TL changes on scale height calculations

The O^+-H^+ transition level is a key input parameter in the above reconstruction procedure and it has a crucial effect on the shape of the topside component profiles. Its effect on the scale height calculations has been investigated by fixing the total electron content and inducing changes (ranging from -30% to +50%) in the transition level around the 'perfect' level provided by the used empirical model. The influence of these TL changes on the O^+ scale height determination is presented in **Fig.5** for day-time (top panel) and night-time (bottom panel) conditions.

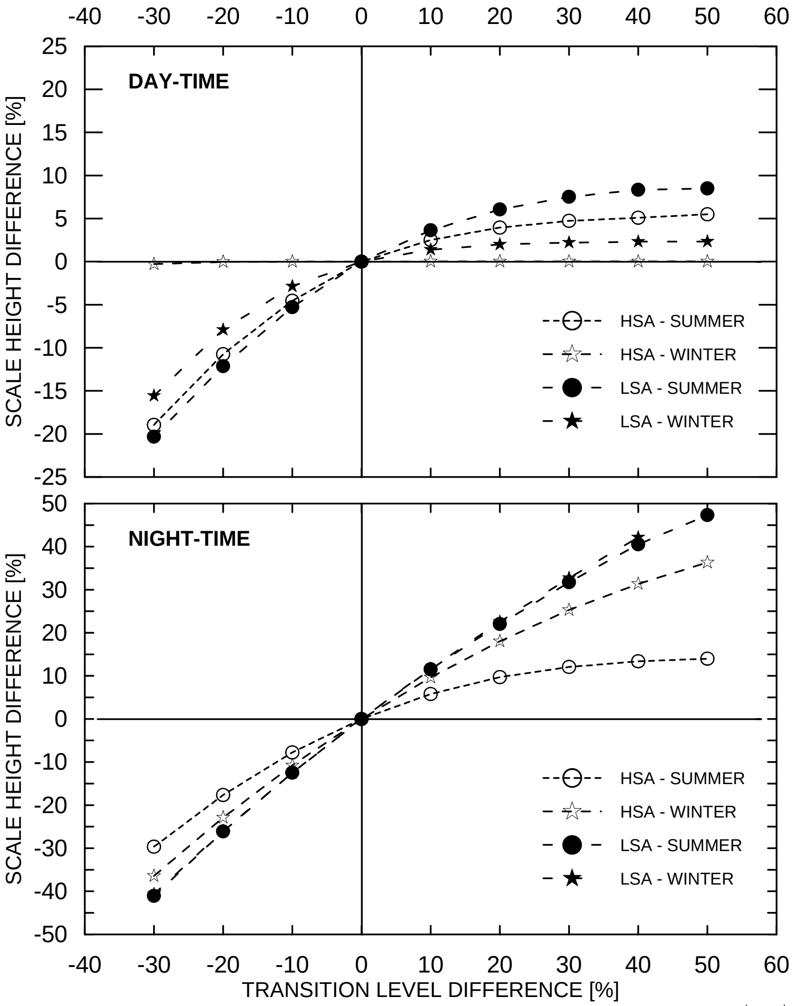


Fig.5 The oxygen ion scale height plotted as a function of the O^+-H^+ transition height for day-time (top panel) and night-time (bottom panel) conditions. The vertical line represents the 'perfect' transition height provided by the empirical model.

Two major conclusions can be drawn.

First, the scale height calculations are much stronger bound to the transition level during night-time than during day-time; a possible reason is the lack of E-layer at night. An error induced via the transition level input will lead to larger error in determining the scale heights during night. The expected error in calculating the scale height due to overestimated transition level will not be larger than 10% in most cases during day-time, but much higher (up to 50%) during night-time.

Second, the scale height gradient decreases when increasing the transition level. This is clearly observed during the day. The transition level cannot be increased indefinitely because at a certain level the scale height determination procedure becomes insensitive to further increase (e.g. HSA winter). This can be explained with the very small part of TEC left to be 'distributed' at altitudes above the TL.

5. Conclusions

The described technique solves a difficult inverse problem - reproduction of the electron density profile from its integral quantity, TEC. At present, routine measurements of electron density rely essentially on ground-based ionosonde soundings which can provide vertical profiles of the bottomside ionospheric electron density only. The offered reconstruction method delivers valuable information about the topside ionospheric and plasmaspheric density based on reliable routine satellite and ionosonde measurements.

Recent developments in monitoring the ionosphere/plasmasphere onboard low-earth orbiting satellites by using signals of global navigation satellite systems such as GPS or GLONASS provide the possibility to further evaluate the reconstruction technique presented here.

The technique should have the capability to be used for evaluating empirical and theoretical models of the ionosphere and plasmasphere systems.

Acknowledgements

This research is supported by the NATO Collaborative Linkage Grant EST.CLG.977103. The ionosonde data were provided by the National Space Science Data Center through the World Data Center-A for Rockets and Satellites, Goddard Space Flight Center.

References

Bilitza D, Rawer K, Bossy L, Gulyaeva T 1993: *Adv. Space Res.*, **13**, 3, 3-13.
Bradley P A, Dudeney J R 1973: *J. Atmos. Terr. Physics*, **35**, 2131.
Chapman S 1963: *J. Geophys. Res.*, **68**, 4, 1174-1174.
Di Giovanni G, Radicella S M 1990: *Adv. Space Res.*, **10**, 11, 27-30.
Dudeney J R 1978: *J. Atmos. Terr. Physics*, **40**, 2, 195-203.
Dudeney J R 1983: *J. Atmos. Terr. Physics*, **45**, 8/9, 629-640.
Jakowski N 1996: in Modern Ionospheric Science, (Eds. H.Kohl, R.Rüster, K.Schlegel), EGS, ProduServ GmbH Verlagsservice, Katlenburg-Lindau, 371-390.
Jakowski N, Sardon E, Engler E, Jungstand A, Klaehn D 1996: *Ann. Geophysicae*, **14**, 1429-1436.
Kutiev I, Stankov S M, Marinov P 1994: *Adv. Space Res.*, **14**, 12, 135-138.
Radicella S M, Leitinger R 2001: *Adv. Space Res.*, **27**, 1, 35-40.
Rawer K 1988: *Adv. Space Res.*, **8**, 4, 191-198.
Sardon E, Rius A, Zarraoa N 1994: *Radio Sci.*, **29**, 577-586.