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RECONSTRUCTION OF THE UPPER ELECTRON DENSITY PROFILE FROM THE OVER-SATELLITE ELECTRON CONTENT

S. M. Stankov

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Abstract

A novel approach for retrieving the topside electron density distribution from space-based observations of the total electron content is presented. By assuming an adequate topside density distribution law, the reconstruction technique utilizes ionosonde and oxygen-hydrogen ion transition level measurements for uniquely determining the unknown ion scale heights and the corresponding ion and electron density profiles. Important applications are envisaged like developing and evaluating empirical and theoretical ionosphere-plasmasphere models.

Key words: electron density reconstruction, total electron content, ion transition level

Introduction. The Total Electron Content (TEC) is defined as the height integral of the electron density from the height of the signal-receiving station, h_r, up to the ceiling height, h_c – height of the signal-transmitting satellite or infinity.

A standard way of measuring TEC is to use ground-based ($h_r = h_g$) receiver processing signals from satellites on geo-stationary orbits like ATS-6, SIRIO; polar orbiting satellites like the US Navy Navigation Satellite System (NNSS), or the Russian Global Navigation Satellite System (GLONASS) satellites; and recently, the Global Positioning System (GPS) satellites. The ground-measured TEC, i.e TEC(h_g, h_c), is denoted gTEC.

Recent developments in the satellite technology allow the signal receiver to be placed onboard a low-earth orbiting satellite, therefore, measuring the over-satellite electron content, i.e. the integral of the electron density from the height of the receiving satellite, h_s ($h_s > h_m$F2), up to the ceiling height, h_c ($h_c \gg h_s$). The over-satellite electron content, TEC(h_s, h_c), is denoted sTEC.

A novel approach has been offered [1] for retrieving the electron density profile from ground-based measurements of the total electron content. This paper presents a method for reconstruction of the vertical electron density distribution from measurements of the over-satellite electron content. Vertical incidence sounding and upper transition level data are also required.

Reconstruction method. This study is focused on the determination of the upper electron profile presented as a sum of its major constituents – the oxygen and hydrogen ion density profiles. Further, the individual ion density distributions are approximated by the hyperbolic secant function in the following manner:

$$N_i(h) = N_i(h_m)\,\mathrm{sech}^2\left(\frac{h-h_m}{2\,H_i}\right), \tag{1}$$

where $N_i(h)$ is the oxygen or hydrogen ion density at height h, H_i is the ion scale height, and $\mathrm{sech}(h) = \cosh^{-1}(h) = 0.5\,(\exp(h) + \exp(-h))$. Therefore, the following 'reconstruction' formula is proposed for calculation of the upper electron density profile:

$$N_e(h) = N_{\mathrm{O}^+}(h_m)\ \mathrm{sech}^2\left(\frac{h-h_m}{2\,H_{\mathrm{O}^+}}\right) + N_{\mathrm{H}^+}(h_m)\ \mathrm{sech}^2\left(\frac{h-h_m}{2\,H_{\mathrm{H}^+}}\right), \quad h > h_m,$$

where $N_e(h)$ is the electron density at height h, H_{O^+} is the O^+ scale height, H_{H^+} is the H^+ scale height, $N_{\mathrm{O}^+}(h_m)$ and $N_{\mathrm{H}^+}(h_m)$ are the O^+ and H^+ densities at height h_m of the F2-layer maximum electron density. Under the assumption that the ionosphere and plasmasphere are isotropic, the scale heights of O^+ and H^+ along the magnetic field lines will have a ratio 1:16 following the definition of the scale height. Also, H^+ is supposed to decrease exponentially from the level of h_m; this is true at altitudes above the $\mathrm{O}^+ - \mathrm{H}^+$ transition height. To obtain the vertical density distribution, the conversion $dh = \sin I ds$ is applied, where dh is the differential element along the vertical, ds is the differential element along the field line, and I is the inclination. If the displacement of the geographic and magnetic poles is ignored, then $dh = \sin[\mathrm{arctg}\,(2\mathrm{tg}\varphi)]\ ds$, where φ is the latitude [2]. Introducing $k = \sin[\mathrm{arctg}\,(2\mathrm{tg}\varphi)]$, the reconstruction formula will read

$$N_e(h) = N_{\mathrm{O}^+}(h_m)\mathrm{sech}^2\left(\frac{h-h_m}{2H_{\mathrm{O}^+}}\right) + N_{\mathrm{H}^+}(h_m)\mathrm{sech}^2\left(\frac{h-h_m}{32kH_{\mathrm{O}^+}}\right), \qquad h > h_m. \tag{2}$$

There are three unknown variables in the proposed formula – the oxygen and hydrogen ion densities at the peak height, i.e. $N_{\mathrm{O}^+} + (h_m)$ and $N_{\mathrm{H}^+}(h_m)$, and the oxygen ion scale height H_{O^+}. These unknowns are determined in the following way.

The over-satellite electron content (see Fig. 1), is the difference between the topside electron content (above h_m) and the electron content enclosed in-between the heights h_m and h_s, i.e.

$$s\mathrm{TEC} = \mathrm{TEC}(h_s; h_c) = \mathrm{TEC}(h_m; h_c) - \mathrm{TEC}(h_m; h_s) = \int_{h_m}^{h_c} N_e(h)dh - \int_{h_m}^{h_s} N_e(h)dh.$$

Both integrals are solved similarly. For the purpose of integration, the following three successive substitutions are applied

$$x = \left(\frac{h-h_m}{2H}\right); \quad \lim_{h\to h_s} x = \frac{h-h_m}{2H}, \qquad \lim_{h\to h_m} x = 0$$

$$y = 2x; \qquad \lim_{x\to x_s} y = \frac{h_s-h_m}{H}, \qquad \lim_{x\to x_m} y = 0$$

$$z = \exp(y); \qquad \lim_{y\to y_s} z = \exp\left(\frac{h_s-h_m}{H}\right), \quad \lim_{y\to y_m} z = 1.$$

Thus, it follows:

$$\int_{h_m}^{h_s} N(h)dh = \int_{h_m}^{h_s} N(h_m)\mathrm{sech}^2\left(\frac{h-h_m}{2H}\right)dh =$$

$$= 2HN(h_m)\int_{x_m}^{x_s}\mathrm{sech}^2(x)dx = 2HN(h_m)\int_{x_m}^{x_s}\frac{4e^{2x}}{(1+e^{2x})^2}dx =$$

$$= 4HN(h_m)\int_{y_m}^{y_s}\frac{e^y}{(1+e^y)^2}dy = 4HN(h_m)\int_{z_m}^{z_s}\frac{1}{(1+z)^2}dz = 4HN(h_m)\left(-\frac{1}{1+z}\right)\Bigg|_{z_m}^{z_s}.$$

Note that the height h_c is practically infinity in the case of GPS measurements since the electron density above the mean height of the plasmapause contributes a negligible quantity to the integrated electron content. Therefore, the result of the integration is

$$(3)\quad \int_{h_m}^{h_s} N(h)dh = 2HN(h_m)\frac{1-\exp\left(\dfrac{h_m-h_s}{h}\right)}{1+\exp\left(\dfrac{h_m-h_s}{H}\right)} \text{ and } \int_{h_m}^{h_c} N(h)dh = 2HN(h_m).$$

Further, after integrating $N_e(h)$ using 'reconstruction' formula (2), and considering the above integral solutions (3), it follows:

$$\begin{aligned}\mathrm{TEC}(h_m,h_c) &= \int_{h_m}^{h_c} N_e(h)dh = \int_{h_m}^{h_c} N_{\mathrm{O}^+}(h)dh + \int_{h_m}^{h_c} N_{\mathrm{H}^+}(h)dh = \\ &= 2H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m) + 2H_{\mathrm{H}^+}N_{\mathrm{H}^+}(h_m) = \\ &= 32kH_{\mathrm{O}^+}N_m + 2(1-16k)H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m)\end{aligned}$$

$$\begin{aligned}\mathrm{TEC}(h_m,h_s) &= \int_{h_m}^{h_s} N_e(h)dh = \int_{h_m}^{h_s} N_{\mathrm{O}^+}(h)dh + \int_{h_m}^{h_s} N_{\mathrm{H}^+}(h)dh = \\ &= 2H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m)X_{\mathrm{O}^+} + 2H_{\mathrm{H}^+}N_{\mathrm{H}^+}(h_m)X_{\mathrm{H}^+} = \\ &= 32kX_{\mathrm{H}^+}H_{\mathrm{O}^+}N_m + 2(X_{\mathrm{O}^+} - 16kX_{\mathrm{H}^+})H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m)\end{aligned}$$

where

$$\begin{aligned}X_{\mathrm{O}^+} &= \left[1-\exp\left(\frac{h_m-h_s}{H_{\mathrm{O}^+}}\right)\right]\Big/\left[1+\exp\left(\frac{h_m-h_s}{H_{\mathrm{O}^+}}\right)\right] \\ X_{\mathrm{H}^+} &= \left[1-\exp\left(\frac{h_m-h_s}{16kH_{\mathrm{O}^+}}\right)\right]\Big/\left[1+\exp\left(\frac{h_m-h_s}{16kH_{\mathrm{O}^+}}\right)\right]\end{aligned}.$$

Hence,

$$\begin{aligned}s\mathrm{TEC} &= \mathrm{TEC}(h_s;h_c) = \mathrm{TEC}(h_m,h_c) - \mathrm{TEC}(h_m,h_s) = \\ &= 2(1-X_{\mathrm{O}^+})H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m) + 2(1-X_{\mathrm{H}^+})H_{\mathrm{H}^+}N_{\mathrm{H}^+}(h_m) = \\ &= 32k(1-X_{\mathrm{H}^+})H_{\mathrm{O}^+}N_m + 2[(1-X_{\mathrm{O}^+}) - 16k(1-X_{\mathrm{H}^+})]H_{\mathrm{O}^+}N_{\mathrm{O}^+}(h_m).\end{aligned}$$

Finally, the over-satellite electron content is

$$\text{(4)} \qquad s\text{TEC} = 4Y_{\text{O}^+}H_{\text{O}^+}N_{\text{O}^+}(h_m) + 4Y_{\text{H}^+}H_{\text{H}^+}N_{\text{H}^+}(h_m) = \\ = 64kY_{\text{H}^+}H_{\text{O}^+}N_m + 4[Y_{\text{O}^+} - 16kY_{\text{H}^+}]H_{\text{O}^+}N_{\text{O}^+}(h_m),$$

where

$$Y_{\text{O}^+} = \exp\left(\frac{h_m - h_s}{H_{\text{O}^+}}\right) \Big/ \left[1 + \exp\left(\frac{h_m - h_s}{H_{\text{O}^+}}\right)\right]$$

$$Y_{\text{H}^+} = \exp\left(\frac{h_m - h_s}{16kH_{\text{O}^+}}\right) \Big/ \left[1 + \exp\left(\frac{h_m - h_s}{16kH_{\text{O}^+}}\right)\right].$$

Expressing $N_{\text{O}^+}(h_m)$ from relation (4), and recalling that $N_{\text{O}^+}(h_m) + N_{\text{H}^+}(h_m) = N_m$ F2, there follows:

$$\text{(5)} \qquad N_{\text{O}^+}(h_m) = (s\text{TEC} - 64kY_{\text{H}^+}H_{\text{O}^+}N_m)/[4(Y_{\text{O}^+} - 16kY_{\text{H}^+})H_{\text{O}^+}],$$

$$\text{(6)} \qquad N_{\text{H}^+}(h_m) = (4Y_{\text{O}^+}H_{\text{O}^+}N_m - s\text{TEC})/[4(Y_{\text{O}^+} - 16kY_{\text{H}^+})H_{\text{O}^+}].$$

Considering the assumed type of topside profile (1), and the fact that the hydrogen and oxygen ion densities are equal at the O^+–H^+ transition level, the next relation holds true:

$$N_{\text{O}^+}(h_m)\text{sech}^2\left(\frac{h_{tr} - h_m}{2H_{\text{O}^+}}\right) = N_{\text{H}^+}(h_m)\text{sech}^2\left(\frac{h_{tr} - h_m}{2H_{\text{H}^+}}\right).$$

Further, after replacing $N_{\text{O}^+}(h_m)$ and $N_{\text{H}^+}(h_m)$ with expressions (5) and (6), the following transcendental equation is constructed:

$$\frac{s\text{TEC} - 64Y_{\text{H}^+}H_{\text{O}^+}N_m}{4(Y_{\text{O}^+} - 16kH_{\text{H}^+})H_{\text{O}^+}N_m}\left\{\text{sech}^2\left(\frac{h_{tr} - h_m}{2H_{\text{O}^+}}\right) + \text{sech}^2\left(\frac{h_{tr} - h_m}{32kH_{\text{O}^+}}\right)\right\} = \text{sech}^2\left(\frac{h_{tr} - h_m}{32kH_{\text{O}^+}}\right).$$

Under the assumption that actual vertical incidence sounding measurements are available, the only unknown variable in this equation is the oxygen ion scale height. Required ionosonde data are the F2-layer critical frequency (foF2), the propagation factor M(3000)F2, and the E-layer critical frequency (foE). Then the F2-layer peak density N_m (also denoted N_mF2) is calculated from the well-known relation

$$Nm\text{F2}[\text{m}^{-3}] = 1.24 \times 10^{10} \times (\,\text{foF2 [MHz]}\,)^2$$

and the F2-layer peak height is estimated [3] by

$$hm\text{F2} = -176 + 1470\,\frac{\text{M}_{3000}\text{F2}\ \left\{(0.0196\ \text{M}_{3000}\text{F2}^2 + 1)\,/\,(1.296\ \text{M}_{3000}\text{F2}^2 - 1)\right\}^{1/2}}{\text{M}_{3000}\text{F2} - 0.012 + 0.253/\,(\text{foF2}/\text{foE} - 1.215)}.$$

The upper transition level h_{tr} is determined from an empirical model [4] based on in-situ satellite measurements.

The unknown oxygen ion scale height is obtained after numerically solving the above transcendental equation. Once the O^+ scale height is found, it is easy to compute the ion densities $N_{O^+}(h_m)$ and $N_{H^+}(h_m)$ using expressions (5) and (6). The upper electron density profile is then recovered by using the reconstruction formula (2).

Results and discussion. The reconstruction method is illustrated using daytime measurements at equinox during high solar activity: over-satellite electron content (sTEC $= 15.2 \times 10^{16}\mathrm{m}^{-2}$), height of satellite ($h_s$ = 450 km), upper transition level (h_{tr} = 1300 km), F2-layer critical frequency (foF2 = 11.0 MHz), E-layer critical frequency (foE = 3.5 MHz), and propagation factor ($\mathrm{M}_{(3000)}\mathrm{F2} = 2.98$) measurements. The recovered topside electron density profile (solid line) is presented in Fig.1; the oxygen and hydrogen ion density profiles are also plotted.

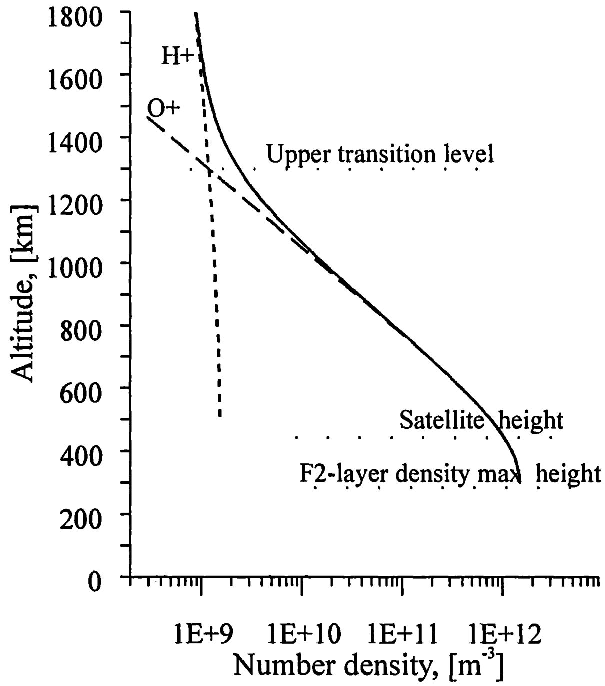

Fig. 1. A reconstructed electron density profile (solid line): topside part obtained after summing up the O^+ (long dashes) and H^+ (short dashes) ion densities

The reconstruction technique has been demonstrated using the secant hyperbolic function as topside ion density profiler. However, the described technique is not restricted to a particular profiler; other analytical models can be used as well, e.g. the 'exponential layer':

$$N(h) = N(h_m)\exp\left(-\frac{h - h_m}{H}\right).$$

In the case when exponential layer is used, the above calculations are carried out in a similar way. This time, the O^+ and H^+ densities at h_m are expressed in the form

$$N_{O^+}(h_m) = \frac{s\mathrm{TEC} - 16kY_{H^+}N_m}{(Y_{O^+} - 16Y_{H^+})H_{O^+}}, \qquad N_{H^+}(h_m) = \frac{Y_{O^+}H_{O^+}N_m - s\mathrm{TEC}}{(Y_{O^+} - 16Y_{H^+})H_{O^+}},$$

where

$$Y_{O^+} = \exp\left(\frac{h_m - h_s}{H_{O^+}}\right), \quad Y_{H^+} = \exp\left(\frac{h_m - h_s}{16kH_{O^+}}\right)$$

and the transcendental equation (O^+ scale height is the variable) acquires the following form:

$$\frac{s\mathrm{TEC} - 16kY_{\mathrm{H}^+}H_{\mathrm{O}^+}N_m}{(Y_{\mathrm{O}^+} - 16kY_{\mathrm{H}^+})H_{\mathrm{O}^+}N_m}\left\{\exp\left(\frac{h_m - h_{tr}}{H_{\mathrm{O}^+}}\right) + \exp\left(\frac{h_m - h_{tr}}{16kH_{\mathrm{O}^+}}\right)\right\} = \exp\left(\frac{h_m - h_{tr}}{16kH_{\mathrm{O}^+}}\right).$$

More complex theoretical profilers like a steady-state theoretical model [5], bound to the F2-layer electron density peak characteristics and the upper transition level, can also be utilized.

Conclusions. The importance of the reconstruction technique should be considered from the following aspects: reliability, flexibility, and applicability.

The reliability of the approach is based on the routine measurements of over-satellite TEC and ground-based ionosonde soundings. Also, the reconstruction procedure employs established, efficient and fast numerical methods.

The approach provides flexibility in terms of structuring, upgrading and testing: choice from various topside profilers, possibility of incorporating the helium ion density in the above equations, convenience when testing the method with different types of measurement data, etc.

Recent advances in monitoring the ionosphere/plasmasphere onboard low-earth orbiting satellites by using signals from global navigation satellite systems (e.g. GPS) offers opportunity to apply the proposed reconstruction technique for both developing and evaluating empirical and theoretical models of the ionosphere – plasmasphere system.

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Geophysical Institute
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 3
1113 Sofia, Bulgaria