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GPS TEC FORECASTING BASED ON AUTO-CORRELATION ANALYSIS

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The GPS-derived TEC has proved to be a robust characteristic representing well the state of the Earth's ionosphere-plasmasphere system during both quiet and disturbed geomagnetic conditions. Successfully forecasting the TEC value can prove invaluable when trying to improve the communications, navigation, and surveying practices. Presented is a new forecasting method based on auto-correlation analysis and consisting of two major parts – first, extrapolation of the TEC monthly medians using Fourier series approximation, and second, geomagnetically-correlated forecast of the TEC relative deviations of from its median value. Preliminary tests show a good agreement between measured and predicted median values. Presented are also important investigations related to the short-term forecast.

Keywords: forecasting; GPS; TEC

1. Introduction

It is well recognised that the strong ionosphere perturbations can cause serious technological problems, for example rapid phase and amplitude fluctuations of satellite signals, range errors, and others. Maps of the basic ionospheric characteristics, such as the F2-layer critical frequency (f_oF2), have been already used for development and evaluation of empirical and theoretical ionosphere-plasmasphere models, for management and optimisation of high-frequency radio and remote sensing systems, etc. (Fox and McNamara 1988, Hanbaba 1998). Being a robust integral characteristic of the ionosphere-plasmasphere system, the Total Electron Content (TEC) has been also successfully used in various investigations of the ionosphere and plasmasphere behaviour under both quiet and disturbed conditions (Jakowski et al. 1990). The investigations became more effective after developing Global Navigation Satellite Systems (GNSS), such as the U.S. Global Positioning System (GPS) and the Russian Global Navigation Satellite Systems (GLONASS) (Jakowski 1996, Davies and Hartmann 1997, Leitinger 1998, Jakowski et al. 1999, Jodogne and Stankov 2002). Considering the ever-growing network of GNSS ground stations, a reliable TEC forecasting model based on GPS observations can be utilized to improve various aspects of the communications, navigation, and geodetic surveying practices.

In the past, several empirical and theoretical approaches have been proposed for simulating and predicting the critical frequency (Hanbaba 1998). Neural networks have been found to be also applicable (Williscroft and Poole 1996). Recently,

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auto- and cross-correlation procedures have been developed for predicting f_oF2 and proving that a reliable short-term forecast should be definitely bound to the geomagnetic activity (Muhtarov and Kutiev 1999, Kutiev et al. 1999, Kutiev and Muhtarov 2001, Muhtarov et al. 2002). The success of the auto-correlation method, together with the observed strong correlation between f_oF2 and TEC (Houminer and Soicher 1996), suggested that the approach is correct and can be utilized, at least to a some extent, in forecasting the TEC value. However, differences are revealed between the f_oF2 and TEC storm-time behaviour (Jodogne and Stankov 2002), e.g. the sustained increase of the TEC relative deviation from the median values during winter-time storms, which complicates the task of TEC forecasting. Additional difficulties are caused by the relatively short GPS TEC time series of less than a full solar cycle; for comparison, the f_oF2 data base consists of many decades of continuous observations.

Presented are preliminary results from developing a new method for GPS TEC forecasting strongly related to the concurrent (and predicted) solar and geomagnetic activity indices and based on extensive auto-correlation analysis. Preliminary tests show a good agreement between measured and predicted median values. Presented are also important investigations related to the short-term forecast. The forecasting is done for a single location; if applied for several locations in a given area, forecast maps can be constructed for this whole area.

2. GPS TEC data base

The development database was built upon the GPS TEC measurements acquired at the German Aerospace Centre (DLR) — Institute of Communications and Navigation (IKN) throughout years 1995 to 2001, therefore covering low, rising and top solar activity. After determining the electron content along a number of ray paths by using a special calibration technique for the ionospheric delay of GPS signals, the slant TEC is mapped to the vertical by using a single layer approximation for the ionosphere at $h_{sp} = 400$ km height. Using the GPS ground stations of the European IGS network, about 60–100 TEC data points are available for reconstructing TEC maps over the area $20°\text{W} \le \lambda \le 40°\text{E}$, $32.5°\text{N} \le \varphi \le 70°\text{N}$. To ensure a high reliability of the TEC maps the measurements are combined with the NTCM2 empirical model (Jakowski 1996). For each grid point value (spacing 2.5°/5° in latitude/longitude) a weighting process between nearest measured values and modeled values is carried out. The achieved accuracy for TEC is in order of $2-3\times10^{16}$ m^{-2} (Jakowski et al. 1996). A linear interpolation algorithm within the corresponding grid pixel is applied to produce a TEC value at an arbitrary location. Interpolations at sites where ionospheric stations exist are important for verification and correlation studies using additional types of observations, and for development of ion/electron density reconstruction techniques (Stankov et al. 2002).

3. GPS TEC dependence on solar and geomagnetic activity

The very existence and dynamics of the Earth's thermosphere-ionosphere-plasmasphere system are determined by the solar and geomagnetic processes (Hargreaves 1992) which are able to inflict strong and multi-faceted responses from the system and its main characteristics f_oF2 and TEC. Systematic studies of these characteristics have been performed for a long time. While the long-term (average) behaviour at a given location is primarily determined by the level of solar activity, the short-term behaviour is additionally influenced by atmospheric and solar-terrestrial conditions, most important being the geomagnetic activity. The focus here is on the GPS TEC, which variability is quite large considering the integral nature of this quantity. Solar and geomagnetic activity effects on GPS TEC will be briefly discussed below after a comment on the activity indicators.

3.1 A note on the solar and geomagnetic indices

A proper choice of indices describing the solar and geomagnetic activity is very important for analysis of the ionosphere-plasmasphere dynamics and successful prediction of its main parameters. The selection of any activity index depends on the purpose of using the index in a particular study and on other factors such as the degree of correlation with the predicted quantity, the index predictability and availability, etc.

Solar indices are used to describe the activity levels of the Sun. The period of solar cycle variations is approximately 11 years, although larger periodicities are also observed. Currently, several solar activity indices are produced and generally available: the Zurich (Wolf) sunspot number R_z, the 12-month running mean of the R_z number, the solar flux at 10.7 cm (2800 mHz) index F10.7, the effective sunspot number SSNe, etc. There is no index which can fulfil all of the above-mentioned requirements. For example, the sunspot number may be the main indicator for long-term trends in solar activity and may possess the longest historical records, but shows serious insufficiencies, such as saturation at high solar activity when observed correlations between index and physical parameters break down. Moreover, the amplitude of the sunspot number has no direct physical meaning. In this study, preferred is the F10.7 index, since it is based on reliable measurements of the radio emission from the Sun at 10.7 cm wavelength. Being a proxy for the background solar ultraviolet radiation flux, the F10.7 index is less noisy than the sunspot number index on a day-to-day basis. In addition, several measurements are made each day in order to remove the effects of short-term radio bursts. Another advantage is its availability in near-real time; also, predictions of the 12-month running mean are widely available.

Geomagnetic indices are used to describe the disturbance levels of the Earth's magnetic field. Many geomagnetic activity indices have been developed during the years: the K_p and A_p planetary indices, the storm D_{st} index, the sub-storm index PC, etc. (Menvielle and Berthelier 1991). The main reasons for developing geomagnetic indices are to quantify variations representative of an isolated effect (e.g. ring current variations described by D_{st}) and to estimate global energy input into the magnetosphere (described by the planetary indices). Most frequently used planetary indices are K_p and a_p, based on 3-hour measurements from 12 ground stations. The values of a_p range from 0 to 400 when expressed in units of 2 nT. K_p is essentially a logarithm of a_p, i.e. $K_p = 1.739 \log(0.423 a_p)$. The daily planetary index A_p is obtained by averaging the 8 values of a_p for each day. K_p and A_p are probably the most suitable indices when carrying out preliminary correlative studies with other geophysical phenomena. A three-day forecast of the A_p index is issued by the World Data Centres, equipping the forecasting method with one of the key input parameters. A standard practice is the forecasted daily value A_p to be nominally assigned to the 1200 LT hour and hourly values (denoted A_{ph}) to be obtained by linear interpolation. The A_{ph} values are then converted to K_{ph} through the above-mentioned empirical relation. For our studies we use the measured K_p and predicted K_{ph}.

3.2 Correlation with solar activity

The solar activity influence on the processes occurring in the ionosphere and plasmasphere has been studied for a long time through observations of their major characteristics. Therefore, several aspects of the ionospheric/total electron content are already well investigated and proved to be significant. One easily detected effect is the increase of the absolute electron content value in step with the increasing solar activity. Although it might be tempting to linearly approximate the above-mentioned relationship, the whole issue is much more complex and needs further analysis, involving the diurnal, seasonal, and spatial behaviour of the electron content.

To demonstrate how the median GPS TEC behaviour correlates with the level of solar activity, provided here are diurnal observations (Fig. 1, presented in TEC units, 1 TECU = 10^{16} m^{-2}) for two different locations, Sofia (42.7°N, 23.4°E) and Juliusruh (54.6°N, 13.4°E), during the low solar activity (LSA) conditions in year 1996 and the high solar activity (HSA) conditions in year 2000. In addition to the diurnal variations, briefly described are also the latitudinal and seasonal variations detected on this particular data set.

Typically, a TEC diurnal profile shows lower values during night and higher values during day. There is usually a pre-sunrise depression, when the daily minimum is observed, and there is also a maximum observed around noon. Of course, there are deviations from this typical diurnal profile as will be shown next.

At LSA, the electron content varies from a night-time minimum between 4.0 and 5.4 TECU up to a day-time maximum between 8.5 and 13.5 TECU. The lowest values are observed during winter and the highest values are observed during summer. In some cases, the high noon values are maintained well into the afternoon, thus forming a mid-day 'plateau', e.g. in April 1996. In other cases (July 1996), a prominent peak appears in the late evening, observed at both stations.

At HSA, the electron content varies from a minimum between 4.6 and 19.3 TECU during the night up to a maximum between 23.2 and 62.4 TECU during the day. The lowest values are again observed during winter but the highest values are

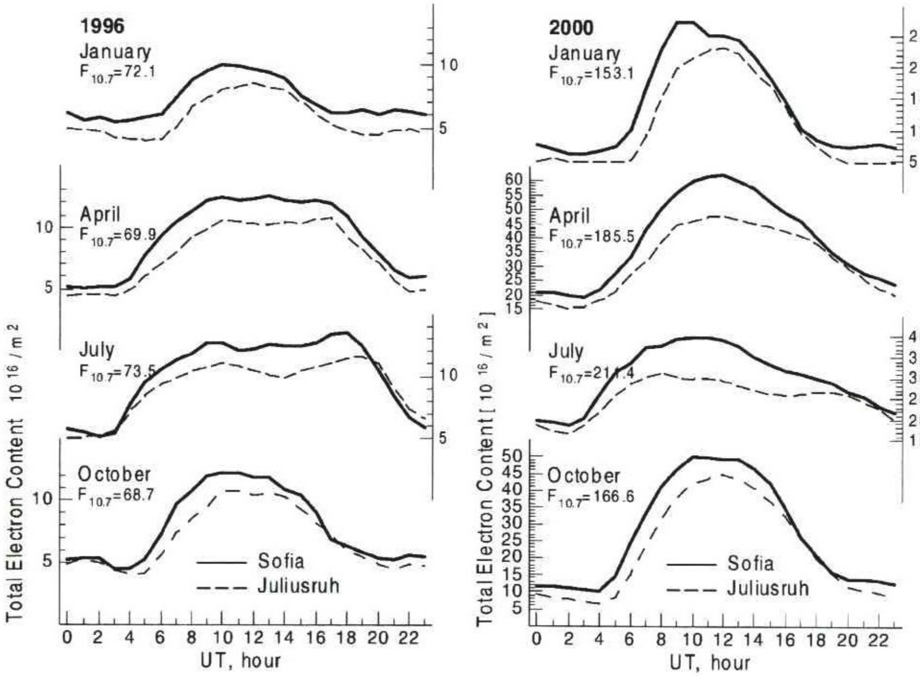

Fig. 1. GPS TEC diurnal behaviour: monthly medians at the sites of stations Juliusruh (54.6°N, 13.4°E) and Sofia (42.7°N, 23.4°E) based on measurements during years of low (1996) and high (2000) solar activity. The daily maximum-to-minimum ratio generally increases with increasing solar activity; for station Sofia the increase in January is from 1.85 up to 4.64, in April – from 2.45 up to 3.23, in July – from 2.60 down to 2.17, and in October – from 2.71 up to 4.93; similarly, for station Juliusruh the increase in January is from 2.13 up to 5.04, in April – from 2.40 up to 3.19, in July – from 2.32 down to 1.90, and in October – from 2.68 up to the record ratio of 6.80

during the near-equinox months of April and October. The pre-sunrise depression is more prominent during HSA, which combined with the higher daytime values leads to a generally increased daily maximum-to-minimum ratio. An exception is the decreased ratio in July 2000 which can be explained with the high geomagnetic activity recorded during this particular month. While the TEC diurnal amplitude at LSA is slightly greater in summer than in winter, it is just the opposite at HSA — the amplitude is much greater in winter. However, the greatest daily maximum-to-minimum ratios are detected in October, at both LSA and HSA.

Although the GPS TEC diurnal behaviour shows some common patterns at both stations, observed are also several differences, clearly demonstrating the expected dependence on latitude. At middle and lower latitudes, and both during LSA and HSA, the GPS TEC increases in equatorward direction. The differences are larger during the day, for example in April the peaks increase by approximately 25% at LSA and 30% at HSA. The occurrences and magnitudes of diurnal maxima are also latitude dependent. For example, notice how the mid-day peak in the HSA summer profile (July, 2000) is significantly eroded at the northern station and a second peak in the late evening hours is formed; an effect attributed to the ionosphere trough.

In order to better analyse the GPS TEC seasonal (annual) behaviour, the running monthly medians have been normalised to the linearly approximated variations over the current calendar year (Fig. 2). Thus, it is clearly shown that the TEC is strongly bound to the solar activity and its variability is significantly increasing with rising solar activity. Other two interesting phenomena are also observed. During day-time, the summer peak observed at LSA is gradually disappearing at higher solar activity; instead, two equinox maxima appear to strongly dominate the annual profile. Night-time, the opposite effect is observed and the summer values are much larger at HSA. At the same time, the winter increase at LSA is not found at HSA.

When analysing solar activity effects, one should not overlook another fact — the delayed response of the Earth's thermosphere-ionosphere-plasmasphere system. Cross-correlation studies involving the ionospheric electron content and the solar radio flux at 10.7 cm wavelength indicate not only a strong correlation but also a response time of the ionosphere to the 27-day solar radiation cycle in the order of 1–2 days (Jakowski et al. 1991). Such a delayed response is important also for a better understanding of the effects of geomagnetic activity.

3.3 Correlation with geomagnetic activity

The GPS TEC dependence on geomagnetic activity is not obvious if using the large time-scale of Fig. 2. Such dependence can be better understood when observations are presented in a smaller time-scale of less than one month (Fig. 3), the storm-time behaviour properly investigated in many case studies, and reliable statistical analysis performed. For the purpose, instead of directly analysing a given ionospheric parameter F (in this case TEC), it could be more helpful to use the relative deviation, F_{rel}, from its median value, F_{med}, that is:

$$F_{\mathrm{rel}} = (F - F_{\mathrm{med}})/F_{\mathrm{med}} \,. \tag{1}$$

A very useful feature of this relative deviation is that it is a dimensionless quantity allowing easier comparison between different parameters and direct comparison between measurements from different locations. Both f_oF2 and TEC relative deviations depend non-linearly on the K_p index. Statistical studies involving f_oF2 show that the average F_{rel} dependence on K_p has a parabolic form (Muhtarov et al. 2002). However, the analysis of GPS TEC data reveals that, while the dependence of the relative GPS TEC on K_{ph} is similar to that of f_oF2 under quiet conditions ($K_{ph} < 4$), the dependence under disturbed conditions is much more complex (Jodogne and Stankov 2002). For example, the GPS TEC relative deviations may be negative for some months (e.g. equinoxes) but there are signs of sustained positive response in the remaining months, most noticeably during winter (Fig. 4). Of course, the GPS TEC time series are not so long comparing to the f_oF2 time series, and the data scattering is rather large, but significant differences are clearly detected. However, further analysis is required.

Such analysis should definitely include the proven delayed ionosphere reaction to the geomagnetic forcing. Previous statistical studies (Kutiev and Muhtarov

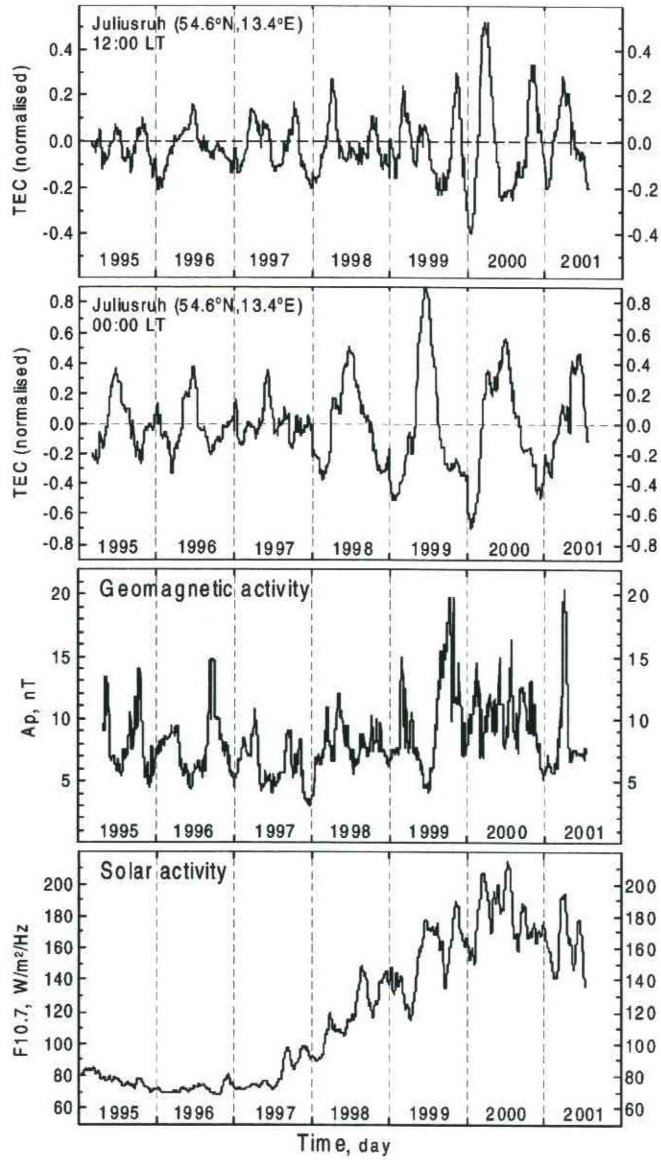

Fig. 2. GPS TEC monthly medians (moving 31-day frame) normalised to the linearly approximated GPS TEC variations over the current calendar year. Data for the site of station Juliusruh (54.6°N, 13.4°E) based on 1995–2001 measurements

2001, Muhtarov et al. 2002) involving the storm-time changes of the critical frequency (f_oF2) and the relative deviation of f_oF2 from its median, i.e. $\Phi = (f_o\mathrm{F2} - f_o\mathrm{F2}_{\mathrm{med}})/f_o\mathrm{F2}_{\mathrm{med}}$, obtained some important results. The cross-correlation between Φ and K_p has been modelled and the delayed reaction of Φ with respect to K_p expressed in terms of the time constant T of their cross-correlation function. Thus,

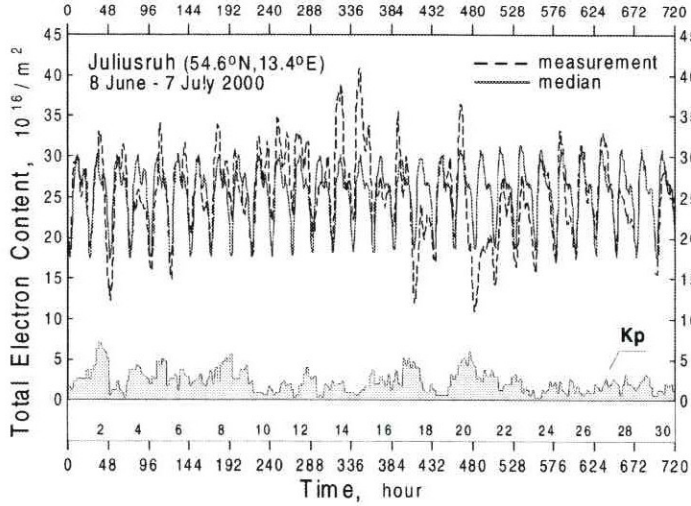

Fig. 3. GPS TEC diurnal behaviour: comparison between actual measurements and median values for the site of station Juliusruh (54.6° N, 13.4° E) obtained during the period 8 June – 7 July 2000

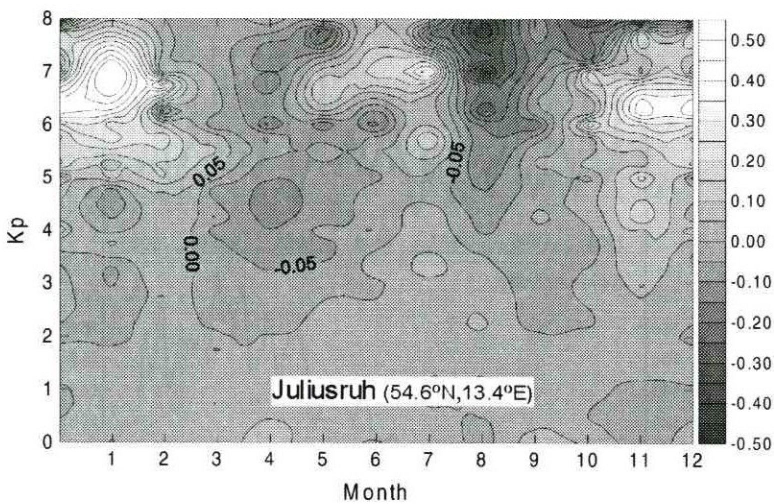

Fig. 4. The average magnitude of the GPS TEC relative deviations from monthly medians, deduced with respect to the geomagnetic activity index K_p (vertical axis) and month of year (horizontal axis). Results obtained for the site of station Juliusruh (54.6° N, 13.4° E) based on 1995–2001 measurements

introduced is a new index K_m (modified K_p), defined as a function $K_m(t)$:

$$K_m(t) = [\exp(1/T) - 1] \sum_{\tau=-\infty}^{t-1} \left\{ \left[K_p^2(\tau) - K_{pa}^2(\tau)\right] \exp\left(-\frac{\tau - t}{T}\right) \right\}, \qquad (2)$$

where $K_{pa}^2(\tau)$ is the corresponding monthly average value of $K_p^2(\tau)$. The variations of the new index K_m closely resembles those of Φ. Actually, $K_m(t)$ linearizes the dependence between Φ and the geomagnetic activity. Such linearization was able to improve the quality of the linear prediction method when applied on f_oF2. In principle, the results are applicable to the GPS TEC relative deviations if using the same method, provided that K_m is estimated with sufficiently enough GPS TEC measurements due to the above-discussed differences between the f_oF2 and GPS TEC reaction to increased geomagnetic activity.

4. Forecasting method

The main idea in the proposed forecasting method is to consider the TEC temporal behaviour as composed of a periodic component and a random component. The periodic component represents the average (annual, diurnal) non-disturbed variation (traditionally represented by monthly medians), while the random component represents the perturbations inflicted on the TEC behaviour due to the strong changes in solar/geomagnetic activity. This approach is well substantiated and clearly demonstrated, especially if you see a comparison between the median and actual GPS TEC variations during a particular month (Fig. 3).

The method consists of two major components: (i) Extrapolation of the TEC monthly median values for up to 15 days ahead. The procedure uses Fourier series approximation based on actual data from the past 12 months and autocorrelation adjustment over the past 30 days of data. (ii) Forecast of the relative deviations of the measured TEC from its median values for up to 24 hours ahead. This forecast depends on the K_p index and is adjusted to the current conditions through an autocorrelation procedure.

4.1 Monthly median forecast

The monthly-median extrapolation procedure uses Fourier series approximation based on actual data from the past 12 months. Behind our reason to base the forecast on the monthly median is the smooth annual variability of the TEC median value for a given hour and the similar pattern exhibited by the TEC measurements at all stations. Measured and approximated TEC values have been compared and showed a very good agreement (Fig. 5). The TEC median value for each hour of the day is separately approximated using the following Fourier series decomposition:

$$TEC(d;h) = \frac{1}{2}a_0(h) + \sum_{i=1}^{n_{\max}} \left[a_i(h)\cos\left(i\frac{2\pi}{360}d\right) + b_i(h)\sin\left(i\frac{2\pi}{360}d\right)\right], \quad (3)$$

where d is the day of the year, h is the local time, and $n_{\max}$ is the number of the harmonic with the highest frequency. The procedure can be used for both interpolation and prediction purposes. If a one-day long gap is placed at the end of the data period, then the method will implicitly extrapolate the values within the imposed gap and will thus offer prediction values up to the length of the gap. In this case less data is required (30 days) to make a reliable one-day prediction. It is justified because the diurnal variations are much more stable than the annual.

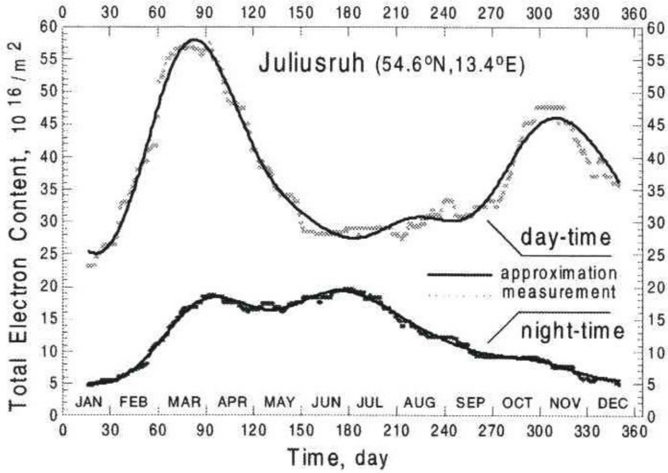

Fig. 5. GPS TEC annual behaviour: comparison between actual measurements (31-day running medians) and Fourier series approximations for the site of station Juliusruh (54.6°N, 13.4°E) obtained during year 2000 for day-time (1200 LT, top) and night-time (0000 LT, bottom) conditions

Both types of extrapolations (annual and diurnal) are used here and the resulting prediction is the average of both values for a given hour.

4.2 Adjustment of the monthly median forecast

The monthly-median extrapolation procedure is improved by an auto-correlation adjustment based on data from the last 30 days. This procedure relies on the diurnal rather than the annual behaviour of TEC. In the presented adjustment procedure, instead of the parameter's absolute value, the relative deviation from its median is used, $F_r = (TEC - TEC_{med})/TEC_{med}$. This function contains no periodic components and can be regarded as a steady-state random process over the considered period of time. Having measurements of F_r at times t_i $(i = 1, 2, ..., n)$, the aim is to predict the values of F at future moments $t_{n+1}, t_{n+2}, \ldots, t_{n+k}$, where (t_n, t_{n+k}) is the length of the prediction period. For the purpose, a regression model (Muhtarov and Kutiev 1999) is used :

$$F(t_{n+1}) = F_m + \sum_{i=1}^{n} \beta_i (F(t_i) - F_m)\,, \tag{4}$$

where β_i are weighting coefficients, and F_m is the sample's median (i.e. for which larger and smaller values are equally probable). The weighting coefficients β_i are determined from the following system:

$$\sum_{i=1}^{n} \beta_i r_{FF}(t_i - t_j) = r_{FF}(t_{n+1} - t_j), \qquad j = 1, 2, \ldots, n\,, \tag{5}$$

where r_{FF} is the auto-correlation function of F. Practically, the real r_{FF} is unknown, so the normalised empirical auto-correlation function is used instead:

$$\varrho_{FF}(\tau) = \frac{\sum F_i F_{j(i)}}{\sqrt{\left(\sum F_i^2\right)\left(\sum F_{j(i)}^2\right)}}, \tag{6}$$

where the summation is performed over the pairs of $F(t)$ having a same time difference τ. It is implicitly assumed that the empirical mean is zero, which is justified for series of several days. Also, because the empirical auto-correlation function is only an estimate of the true auto-correlation function, the accuracy of such estimate depends on the size of the data sample, which should be much larger than the number of the unknown coefficients.

4.3 Short-term prediction

The short-term forecast of GPS TEC is further improved by introducing cross-correlation between the TEC relative deviation from its median value and the geomagnetic index. In this way, it is implicitly assumed (although not completely accurate) that the geomagnetic activity is the sole reason for ionospheric disturbances.

The following regression formula (Muhtarov et al. 2002) is proposed for predicting the relative deviation:

$$F_{n+1} = F_m + \sum_{i=1}^{n} \beta_i (F_i - F_m) + \sum_{i=1}^{n+1} \gamma_i (G_i - G_m), \tag{7}$$

where γ_i are weighting coefficients, $G(t)$ is the geomagnetic function, and G_m is the median value of the sample $\{G_i,\ i = 1, \ldots, n\}$. Principally, the geomagnetic function can be any index satisfactorily measuring the geomagnetic activity, for example A_p, K_p, or K_{ph} (even K_m). However, the statistical theory teaches that the mutual correlation between two random variables, such as F and G, is highest when they are linearly dependent. Unfortunately, it has been proven (Fig. 4) that the dependence between F and K_p is highly non-linear. Therefore, to ensure a better prediction, K_p is not used directly but a function which linearizes the relationship between F and K_{ph}. Thus, the geomagnetic function $G(t)$ is actually the conditional expectation of F with respect to K_{ph}. It should be stressed that the discussed geomagnetic function strongly depends on the location of the GPS TEC station and should be estimated over a long-term series of measurement data.

All weighting coefficients are now determined from the following system:

$$\sum_{i=1}^{n} \beta_i \varrho_{FF}(\tau_{ij}) + \frac{\sigma_G}{\sigma_F} \sum_{i=1}^{n+1} \gamma_i \varrho_{FG}(\tau_{ij}) = \varrho_{FF}(\tau_{n+1,j}), \quad j = 1, 2, \ldots, n, \tag{8}$$

$$\sum_{i=1}^{n} \beta_i \varrho_{GF}(\tau_{ij}) + \frac{\sigma_G}{\sigma_F} \sum_{i=1}^{n+1} \gamma_i \varrho_{GG}(\tau_{ij}) = \varrho_{GF}(\tau_{n+1,j}), \quad j = 1, 2, \ldots, n+1. \tag{9}$$

The auto-correlation functions depend on the time shift, $\tau_{ij} = t_i - t_j$, only. Here, ϱ_{FG} (note that $\varrho_{FG} = \varrho_{GF}$) is the cross-correlation function between $F(t)$ and $G(t)$, defined

$$\varrho_{FG}(\tau) = \frac{\sum F_i G_{j(i)}}{\sqrt{\left(\sum F_i^2\right)\left(\sum G_{j(i)}^2\right)}}, \tag{10}$$

where again the summation is performed over the pairs of $F(t)$ having the same time difference τ in the data sample. The F and G auto-correlation functions are ϱ_{FF} and ϱ_{GG}, and the corresponding standard deviations are:

$$\varrho_F = \sqrt{\frac{1}{n}\sum_{i=1}^{n} F(t_i) - \left(\frac{1}{n}\sum_{i=1}^{n} F(t_i)\right)^2}, \tag{11}$$

$$\varrho_G = \sqrt{\frac{1}{n+1}\sum_{i=1}^{n+1} G(t_i) - \left(\frac{1}{n+1}\sum_{i=1}^{n+1} G(t_i)\right)^2}. \tag{12}$$

Statistical sufficiency of the auto-correlation functions is ensured by using parametric expressions. The auto-correlation function of G decreases exponentially with increasing the time difference, τ, which leads us to the conclusion that this auto-correlation function has the following form:

$$R_{GG}(\tau) = G_{\text{mean}} + \sigma_G^2 \exp(-|\tau|/T_G) \tag{13}$$

known in the signal-processing theory as 'random telegraphic wave'. The auto-correlation is symmetrical with respect to $\tau = 0$ and decreases with a time constant T_G.

4.4 Procedure

In resume, the geomagnetically-correlated forecasting procedure works as follows. First, the TEC data, from the 30-day period prior to the date of forecast, are assembled and the TEC hourly medians are calculated. Second, all TEC relative deviations from the median value are computed. Third, the K_{ph} index is obtained over the whole data period including the day of the prediction. Next, all values G_i of the geomagnetic function are derived. Further, based on the available data from the 30-day period, the cross- and auto-correlation functions of F and G are determined together with the weighting coefficients. Finally, the required 24 values of F_r for the forecasted day are obtained using the regression formula.

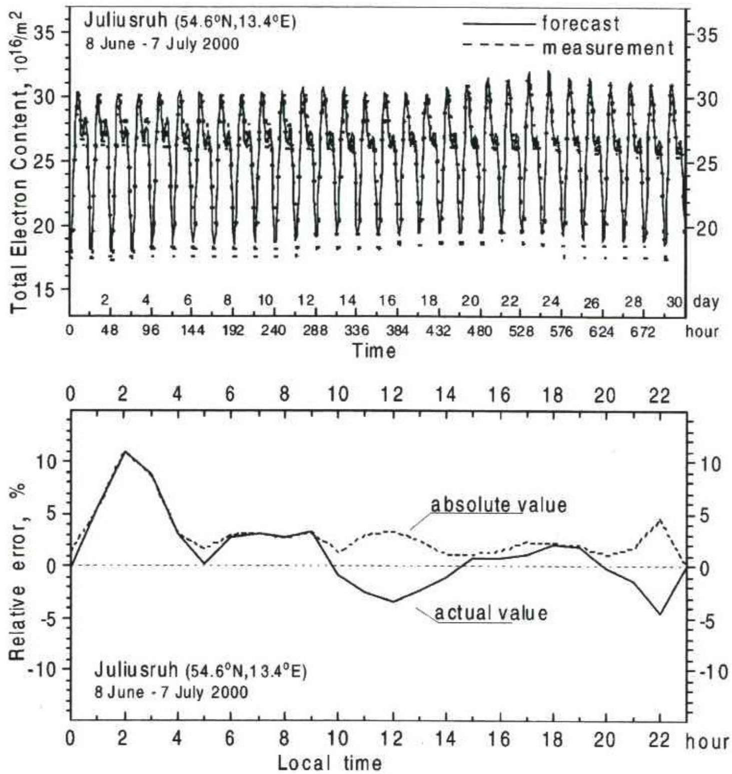

Fig. 6. TEC monthly median forecast (24-hour): measured and forecasted values (top panel) and relative errors (bottom panel), for the site of station Juliusruh (54.6° N, 13.4° E)

5. Results and discussion

The TEC hourly time series is considered as a sum of two components — periodic and random. The periodic component is non-random and describes the TEC mean behaviour (represented here by the 31-day running medians). On the other hand, the random component describes the TEC fluctuations supposedly inflicted by the geomagnetic field disturbances. These fluctuations are supposed to be a manifestation of a stationary stochastic process. The stationarity hypothesis implies that the mean $E\{n(t)\}$ and the product moment $E\{n(t)n(t+\tau)\}$ are independent on t, where E denotes the mathematical expectation. Such interpretation suggests that the TEC median behaviour is the signal, and the fluctuations are noise. The method has been tested for 24-hour median predictions; exemplary test results are provided in Fig. 6 for high solar activity. The first prediction hour is 0000 LT and the last 2300 LT of the same day. In the top panel, the averaged values of the diurnal and annual extrapolation is given. In the bottom panel, the absolute and real values of the relative error (averaged within each hour of the day) are plotted. The relative errors are larger during night (reaching occasionally 10%) but in the rest of the time are varying between 1 and 3%. The results of the geomagnetically-correlated short-term prediction of ionospheric parameters are presented and discussed elsewhere (Muhtarov et al. 2002) and will not be repeated here.

Basic sources of errors in this forecasting method are: the insufficiency of data measurements within a certain area, inevitable data smoothing inherent in the mathematical formulation, unknown and unstable relationship between the ionospheric characteristic and the solar-geomagnetic activity, etc.

6. Summary and conclusions

A new method for monthly median and short-term prediction of TEC has been presented. First, the TEC monthly medians are extrapolated using Fourier series approximation based on actual data from the past 12 months and autocorrelation adjustment over the past 30 days of data. Second, the TEC relative deviation from its median is extrapolated for up to 24 hours ahead using geomagnetically-correlated regression model. The advantages of the presented approach are in the using of routine reliable measurements, employing established numerical methods, offering real-time forecasting capabilities, etc. Therefore, this TEC short-term forecasting can be a powerful instrument for both research and operating practice and might be considered in many space-weather related applications.

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